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Return Predictability using an Endogenous Regime Switching Model *

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Abstract This paper examines whether stock excess return predictability is dependent upon the stock market volatility. The paper introduces a two-state regime switching model with endogenous feedback effect for the stock return predictability test. To model regime switching, this paper adopted a new approach proposed by Chang *et al.* (2017), allowing an endogenous feedback effect channel through which the underlying time series affect the next period volatility regime. This paper shows that modeling such a channel is important in the return predictability context to incorporate the leverage effect. Monte Carlo simulation results demonstrated that additional power gain and bias improvement could be achieved in the endogenous regime swithcing (ERS) model, compared to the conventional Markov switching model. The empirical test results using the ERS model indicate that none of the tested predictors have significant predictive power when stock returns are highly volatile. However, the dividend-price ratio and macro variables such as T-bill rate and term spread had significant predictability, at least in the low volatility regime.

Keywords Predictive regression, Regime switching model, Endogenous feedback effect, Time-varying volatility

JEL Classification C32, C50, G12

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1. INTRODUCTION

Is stock excess return predictable? As noted by Phillips and Lee (2013), the efficient market hypothesis implies that stock prices have a martingale property, making stock returns unpredictable. However, return predictability has been a controversial issue in the literature, since some empirical evidence has been suggested claiming that stock return in predictable. Since Campbell and Shiller (1988) argued that the relationship between fundamental value and asset price might allow the fundamental to price ratio to predict stock returns, much subsequent literature has considered price ratios, such as dividend-price ratio and earning-price ratio, as predictors for stock returns including Lewellen (2004), Cochrane (2008), Campbell and Yogo (2006),and Welch and Goyal (2008) among many others.

However, lots of previous literature reports that return predictability is not stable. According to Ang and Bekaert (2007) and Lettau and Ludvigson (2005), the return predictability seems unstable to the inclusion of the period from the mid-1900s. Welch and Goyal (2008) demonstrated that in-sample predictability fitting is consistently better than the out-of-sample predictability forecasting performance, which can be partly explained by time-varying predictive relations ass noted by Paye and Timmermann (2006). Without proper consideration for time-varying relationship between return and forecasting variable, full-sample inference on predictability might yield biased forecasts as noted in Viceira (1997), Schaller and Norden (1997), Paye and Timmermann (2006), Lettau and Nieuwerburgh (2005), Pettenuzzo and Timmermann (2011) and Hammerschmid and Lohre (2017).

In this paper, time-varying nature of return predictability was investigated using a two-state volatility regimes switching model that can accommodate a market volatility dependent predictability. Specifically, we use an approach using an autoregressive latent factor proposed by Chang *et al.* (2017). In fact, Yang *et al.* (2019) investigated a GARCH model adopting the the endogenous regime switching model. Unlike the conventional Markov switching model, which assumes that the current state is determined only by the past states, this new approach also allows underlying time series to be correlated with the next period latent factor, allowing return series to have an endogenous feedback effect on the next period volatility regime. We also show that such a channel needs to be considered in the return predictability context to incorporate the leverage effect. Yang *et al.* (2019) Our Monte Carlo simulation shows that additional information from the underlying time series allows a sharper inference of state process, resulting in power gain and bias improvement compared to the conventional Markov switching model.

It is well-known that many predictors are highly persistent and that their innovations are correlated with those of returns. If so, as Stambaugh (1999) noted, the limit distribution of test statistics can be non-standard, resulting in over-rejection of true null when the ordinary least squares regression is conducted. Moreover, least square estimates would suffer severe finite sample bias when innovations of returns and predictors are strongly correlated. By jointly estimating the process of return and predictor series with structure given to their innovation correlations, the ERS model could alleviate the over-rejection problem and finite sample bias.

Empirical test results in this paper indicate that none of the tested variables offer significant predictability in the high volatility regime. However, in the low volatility regime, the dividend-price ratio and macro variables such as T-bill rate and term spread did offer significant predictability for stock returns. When exogenous shock strikes the stock market, stock prices would change rapidly, making the stock market enter the high volatility regime. Under such a circumstance, it would be extremely hard to predict stock returns using any past information, due to unpredictable exogenous shock. We also find that the stock excess return volatility of the high volatility regime is more than three times that of the low volatility regime. Moreover, endogeneity parameters are also estimated to have significantly large negative value. This goes well with the leverage effect; a negative shock on the current return increases the next period volatility.

Monte Carlo simulation is also conducted to examine what happens when either volatility regimes or the endogenous feedback effect is ignored. We show that the size distortion and finite sample bias in the estimation of key parameters were bigger when volatility regimes were ignored. We also show that the bias reduction of the ERS model tends to be more substantial when the regime switching parameter has large switching values.

The remainder of this paper is as follows. Section 2 illustrates the models used for empirical analysis. Section 3 describes how to estimate the ERS model in detail. In Section 4, an empirical analysis using real data is presented. Section 5 provides Monte Carlo evidence that the ERS model outperforms the conventional Markov switching models regarding size, power, and bias. Section 6 presents the conclusion.

2. THE MODEL

2.1. A REGIME SWITCHING MODEL WITH ENDOGENOUS AUTOREGRESSIVE LATENT FACTOR

The two-state regime switching model was adopted to capture switching stock return predictability with its volatility regimes. The state characterizing rule was set as $\sigma_u(s_t = 0) < \sigma_u(s_t = 1)$, allowing the state 1 to be a high volatility regime and the state 0 to be a low volatility regime. The predictive regression parameters, α and β, were also modeled to have switching values given volatility regimes.

$$y_t = \alpha(s_t) + \beta(s_t)x_{t-1} + \sigma_u(s_t)u_t \tag{1}$$

When $\beta(s_t)$ is not equal to zero, the univariate predictor x_{t-1} has some predictive power on future stock excess returns y_t. Therefore, $\beta(s_t)$ is a parameter of interest indicating whether the predictor has any predictive power over stock excess return. By doing so, this model provides a way to compare the predictive power of a predictor in high $(\overline{\beta})$ and low $(\underline{\beta})$ volatility regimes.[1] The predictor was also assumed to have regime switching volatilities.[2]

$$x_t = \mu + \phi x_{t-1} + \sigma_v(s_t)v_t \tag{2}$$

where u_t and v_t are normalized errors with a variance of one.

$$u_t = \pi v_t + \sqrt{1-\pi^2}\varepsilon_t \tag{3}$$

where v_t and ε_t are independent with a normalized variance of one. Therefore, π denotes contemporaneous correlation between the innovation of y_t and that of x_t. As can be shown in (2) and (3), this model give room for persistence in (x_t) and innovation correlation π to be structured within the model.

This paper implemented an innovative approach developed by Chang *et al.* (2017) to model regime switching. According to their new approach, regimes are determined by whether the autoregressive latent factor (f_t) exceeds the threshold level or not.

$$s_t = 1\{f_t \geq \tau\} \tag{4}$$

[1] The underline notation is for the parameter value in the low volatility regime while the overline notation is for the parameter value in the high volatility regime.

[2] Among predictors considered in this paper, price ratios (dividend-price ratio, earning-price ratio and smoothed real earnings-real price ratio) were assumed to have regime switching volatility while macro variables such as treasury bill rate and term spread were assumed not. The grounds for such setting will be elaborated on Section 4.2.

$$f_{t+1} = \lambda f_t + \eta_{t+1} \tag{5}$$

where $s_t = \{0, 1\}$, which is determined by latent factor f_t and threshold level τ. The strength of their new method comes from the correlation between the next period innovation of latent factor η_{t+1} and underlying time series innovations u_t and v_t. With a latent factor that is correlated with previous underlying time series, regime could be determined endogenously in this new approach.

$$\begin{pmatrix} v_t \\ \varepsilon_t \\ \eta_{t+1} \end{pmatrix} =_d N \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & \rho_1 \\ 0 & 1 & \rho_2 \\ \rho_1 & \rho_2 & 1 \end{pmatrix} \right) \tag{6}$$

The current shock on return series may affect next period latent factor and volatility regime since u_t and η_{t+1} are correlated by $\pi\rho_1 + \sqrt{1-\pi^2}\rho_2$. By allowing underlying time series to affect the next period latent factor and volatility regime determination, new approach relieves the assumption of a conventional Markov switching (CRS, i.e., conventional regime switching) model that state is determined independently from the underlying time series. Such an endogenous feedback effect of the underlying time series is an important extension the ERS model has made compared to the CRS model.

The ERS model is a natural extension of the conventional Markov regime switching (CRS) model. Chang *et al.* (2017) showed that an ERS model reduces to the CRS when the autoregressive latent factor is exogenous and stationary. Therefore, the likelihood ratio test (LR test) can be conducted to check whether the likelihood of the unrestricted model (ERS) is significantly higher than that of the restricted model (CRS).

The CRS model can be easily formulated if we impose restriction in equation (5) and (6) as follows:

$$|\lambda| < 1$$

$$\begin{pmatrix} v_t \\ \varepsilon_t \\ \eta_{t+1} \end{pmatrix} =_d N \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right)$$

As we can see the model, the ERS model can easily accommodate the leverage effect by considering the possibility of negative ρ_1 or ρ_2 that implies a negative shock on stock returns is likely to increase volatility in the next period.

Another traditional way of modeling the stock return predictability is to use a simple predictive regression model as follows:

$$\begin{aligned} y_t &= \alpha + \beta x_{t-1} + u_t \\ x_t &= \mu + \phi x_{t-1} + v_t \end{aligned} \tag{7}$$

where u_t and v_t are i.i.d. innovations with variance σ_u^2 and σ_v^2, respectively. A typical way to analyze the traditional model is using a least squares estimate $\widehat{\beta}$ and its t-ratio. However, some characteristics of series (y_t) and (x_t) might cause size distortion and finite sample bias in $\widehat{\beta}$. Stambaugh (1999) argued that the OLS estimate of β is biased, and the null of no predictability $(\beta = 0)$ is over-rejected when stock return innovation (u_t) is strongly correlated with that of predictor (v_t) and the predictor is highly persistent. It is important to properly manage such problems, because most predictor series are highly persistent and some of them have innovations strongly correlated with that of stock returns. The ERS model jointly estimate two equations with structure given to innovations u_t and v_t, and can alleviate size distortion and finite sample bias as shown in our simulation results in Section 4.

2.2. THE ESTIMATION OF THE ERS MODEL

For the maximum likelihood estimation of the model, the log-likelihood function can be written as

$$\ell(y_1, \cdots, y_n, x_1, \cdots, x_n) = \log p(y_1, x_1) + \sum_{t=2}^{n} \log p(y_t, x_t | \mathscr{F}_{t-1}) \tag{8}$$

where $\mathscr{F}_t = \sigma((x_s)_{s \le t}, (y_s)_{s \le t})$ is a set of information given for $t = 1, \cdots, n$. There is a set of unknown parameters $\theta \in \Theta$. In this case, θ is a vector containing paramters like $\alpha(s_t)$, $\beta(s_t)$, $\sigma_u(s_t)$, μ, ϕ, $\sigma_v(s_t)$, π, λ, τ, ρ_1 and ρ_2 in (2)-(6).

By allowing an endogenous feedback effect as in (6), the state process is not only influenced by the previous states but also by the underlying time series. Therefore, state process (s_t) alone is not a first-order Markov process; rather, (s_t, y_t, x_t) on $\{0,1\} \times \mathbb{R} \times \mathbb{R}$ is a first-order Markov process. This makes the conventional Markov switching filter not applicable, resulting in a need for a modified Markov switching filter as suggested in Chang *et al.* (2017). They showed that a modified Markov switching filter should be developed to model the endogeneity channel, through which the underlying time series affect the next period latent factor in the ERS Model. Here, we skipped the detailed derivation of the modified Markov switching filter to space space.[3]

3. EMPIRICAL ANALYSIS

This section tests whether each predictor has predictive power for stock excess returns. Three models are used to test predictability of each predictor: The traditional predictive regression model (PRM) in equation 7, the conventional Markov regime switching model and the endogenous regime switching model introduced in the previous section. As potential predictors, four individual variables including price ratios such as the dividend-price ratio and earning-price ratio and macro variables such as T-bill rate and term spread are considered.

3.1. DESCRIPTION OF DATA

The full sample period is from January 1926 to December 2016. For stock market returns, monthly *value-weighted return including distributions* (VWRETD) from the Center for Research in Security Prices (CRSP) was used. The monthly excess return was computed by subtracting risk-free rates from stock market returns. The 3-month T-bill rate was used as a risk-free rate.

There are two macro variables considered as potential predictors for stock excess returns: the 3-month T-bill rate and the term spread. As the 3-month T-bill rate, *3-Month Treasury Bill: Secondary Market Rate* from FRED was used from 1934. But before, *U.S. Yields On Short-Term United States Securities, Three-Six Month Treasury Notes and Certificates, Three Month Treasury* from NBER Macrohistory database was used since 1934. The term spread is a difference between the long term yield and the T-bill rate. As a long term yield, *Long-Term Government Bond Yield* from Ibbotson's *Stocks, Bonds, Bill and Inflation Yearbook* was used. This dataset was obtained from the 2016 updated version of one used in Welch and Goyal (2008), uploaded to the webpage of Amit Goyal.[4]

In addtion, two more price ratios as predictors were considered: the dividend-price ratio and earning-price ratio. According to Campbell and Shiller (1988), the dividend-price ratio(or earning-price ratio) was calculated as a ratio of dividends(or earnings) over the past year relative to current price. This data was offered by *U.S. Stock Markets 1871-Present and CAPE Ratio*, uploaded in the

[3] The detailed formulation of the modified Markov switching filter is presented in Appendix that is available upon request from authors.

[4] http://www.hec.unil.ch/agoyal/

online data of Robert Shiller.[5] For the actual predictive regression, the natural logarithm was taken on price ratios.

3.2. ESTIMATION RESULTS

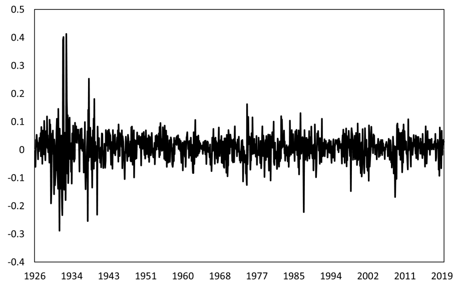
The estimation results using four potential predictors for the predictability test are presented in Section 3.2. Each predictor was tested individually, and estimation results using three models including LIN, CRS and ERS were compared.

As explained in the previous sections, we use a two-state regime switching model. The two states in the model, $\sigma_u(s_t = 0) < \sigma_u(s_t = 1)$, implies that stock return volatility is higher in state 1 than in state 0. Among tested predictors for stock returns, price ratios are highly likely to have a similar volatility pattern with stock returns since all of them were divided by stock price. Therefore, it can be presumed that price ratios also have high volatility in state 1, when stock return volatility is high. To reflect these presumptions, when price ratios are used as an individual predictor for stock returns, a regime was also given for predictor volatility; $\sigma_v(s_t = 0) < \sigma_v(s_t = 1)$ as presented in Table 2. On the other hand, macro variables are less likely to have higher volatility whenever stock return is in the high volatility regime. Therefore, macro variable volatilities were not assumed as regime switching parameters, that is, the estimation of σ_v is not dependent upon s_t as in Table 1.

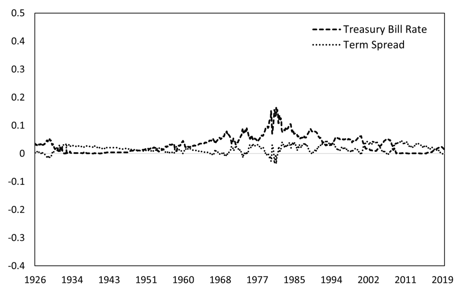
[5]http://www.econ.yale.edu/~shiller/data.htm

Figure 1: Time Series Plot

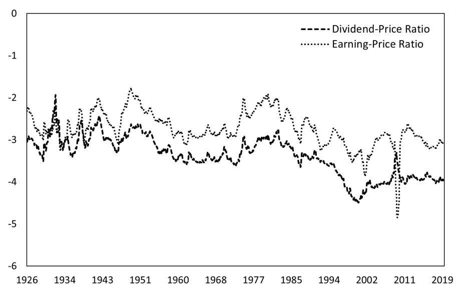
(a) Stock Excess Return



(b) Macro Variable



(c) Fundamenetal to Price Ratio



A time series plot of the variables used in this analysis for the sample period 1926/01-2016/12. Figure 1-(a) plots stock excess returns calculated using VWRETD from CRSP and a 3-Month T-bill rate. Figure 1-(b) plots macro variables: 3-Month T-bill rate and term spread, which is the difference between long term yield on government bond and the 3-Month T-bill rate. Figure 1-(c) plots price ratios: log dividend-price ratio and log earning-price ratio.

Table 1: Estimation Result with Macro Variable

Panel A. T-bill Rate

	OLS	CRS		ERS	
		$s_t = 0$	$s_t = 1$	$s_t = 0$	$s_t = 1$
α	0.0100	0.0141	-0.0062	0.0137	-0.0028
	(0.0024)	(0.0020)	(0.0129)	(0.0019)	(0.0128)
β	-0.1000	-0.1307	-0.6381	-0.1288	-0.7924
	(0.0527)	(0.0406)	(0.5182)	(0.0401)	(0.5256)
σ_u		0.0384	0.1172	0.0382	0.1185
		(0.0010)	(0.0088)	(0.0010)	(0.0092)
μ	0.0002	0.0002		0.0002	
	(0.0002)	(0.0002)		(0.0002)	
ϕ	0.9932	0.9932		0.9933	
	(0.0035)	(0.0035)		(0.0035)	
σ_v		0.0036		0.0036	
		(0.0001)		(0.0001)	
π		-0.1334		-0.1352	
		(0.0300)		(0.0301)	
λ		0.9949		0.9922	
		(0.0043)		(0.0057)	
τ		11.7118		9.7671	
		(4.7266)		(3.6476)	
ρ_1				0.2300	
				(0.1828)	
ρ_2				-0.9567	
				(0.0389)	
log likelihood		6634.5612		6638.5002	
p-value for LR test		0.0195			

Panel B. Term Spread

	OLS	CRS		ERS	
		$s_t = 0$	$s_t = 1$	$s_t = 0$	$s_t = 1$
α	0.0039	0.0055	-0.0204	0.0052	-0.0156
	(0.0026)	(0.0020)	(0.0229)	(0.0020)	(0.0220)
β	0.1645	0.2183	0.3676	0.2205	0.1996
	(0.1238)	(0.1009)	(0.9268)	(0.0981)	(0.8762)
σ_u		0.0386	0.1171	0.0384	0.1183
		(0.0010)	(0.0086)	(0.0010)	(0.0090)
μ	0.0006	0.0006		0.0006	
	(0.0002)	(0.0002)		(0.0002)	
ϕ	0.9619	0.9621		0.9621	
	(0.0081)	(0.0081)		(0.0081)	
σ_v		0.0036		0.0036	
		(0.0001)		(0.0001)	
π		0.0389		0.0354	
		(0.0304)		(0.0306)	
λ		0.9954		0.9926	
		(0.0039)		(0.0054)	
τ		12.3974		10.0862	
		(5.0092)		(3.7694)	
ρ_1				-0.1973	
				(0.2899)	
ρ_2				-0.8932	
				(0.1877)	
log likelihood		6635.1620		6638.7150	
p-value for LR test		0.0286			

Notes: Monthly data from 1926 to 2019 was used. Standard errors are in parenthesis. The LR test was conducted with the null of no endogeneity $(\rho_1 = \rho_2 = 0)$.

Table 2: Estimation Result with Fundamental to Price Ratio

	Panel A. Dividend-Price Ratio						Panel B. Earning-Price Ratio				
	OLS	CRS		ERS			OLS	CRS		ERS	
		$s_t=0$	$s_t=1$	$s_t=0$	$s_t=1$			$s_t=0$	$s_t=1$	$s_t=0$	$s_t=1$
α	0.0297	0.0281	0.0383	0.0280	0.0870	α	0.0274	0.0189	0.0020	0.0190	-0.0178
	(0.0119)	(0.0095)	(0.0419)	(0.0094)	(0.0422)		(0.0108)	(0.0092)	(0.0411)	(0.0090)	(0.0378)
β	0.0068	0.0055	0.0109	0.0057	0.0256	β	0.0075	0.0035	0.0018	0.0038	-0.0056
	(0.0035)	(0.0028)	(0.0131)	(0.0027)	(0.0133)		(0.0039)	(0.0034)	(0.0137)	(0.0033)	(0.0126)
σ_u		0.0364	0.1088	0.0364	0.1129	σ_u		0.0367	0.1139	0.0366	0.1146
		(0.0009)	(0.0061)	(0.0009)	(0.0066)			(0.0010)	(0.0069)	(0.0009)	(0.0068)
μ	-0.0151	-0.0110		-0.0129		μ	-0.0291	-0.0151		-0.0150	
	(0.0099)	(0.0078)		(0.0078)			(0.0118)	(0.0082)		(0.0083)	
ϕ	0.9958	0.9977		0.9973		ϕ	0.9897	0.9954		0.9955	
	(0.0029)	(0.0023)		(0.0023)			(0.0043)	(0.0030)		(0.0030)	
σ_v		0.0303	0.0927	0.0301	0.0934	σ_v		0.0328	0.1403	0.0325	0.1387
		(0.0008)	(0.0051)	(0.0008)	(0.0052)			(0.0010)	(0.0087)	(0.0009)	(0.0082)
π		-0.6434		-0.6430		π		-0.5464		-0.5483	
		(0.0183)		(0.0185)				(0.0220)		(0.0218)	
λ		0.9775		0.9626		λ		0.9790		0.9778	
		(0.0096)		(0.0144)				(0.0098)		(0.0089)	
τ		4.9243		3.8996		τ		5.1664		5.0709	
		(1.1447)		(0.8203)				(1.2697)		(1.1019)	
ρ_1				0.0689		ρ_1				0.2019	
				(0.1212)						(0.1660)	
ρ_2				-0.9697		ρ_2				-0.7635	
				(0.0988)						(0.1131)	
log likelihood		4295.2361		4312.2767		log likelihood		4037.5996		4048.2526	
p-value for LR test		3.9751E-08				p-value for LR test		2.3629E-05			

Notes: Monthly data from 1926 to 2019 was used. Standard errors are in parenthesis. The LR test was conducted with the null of no endogeneity $(\rho_1 = \rho_2 = 0)$.

3.2.1 Return Predictability with Macro Variables

According to Chen (1991), macro variables such as T-bill rate or term spread can be used to predict asset returns since they can provide prospects for the future economy, which affects the asset market and thereby asset returns. There are many subsequent literatures which supported predictability of macro variables for stock returns: Chen (1991), Fama and French (1989), Campbell and Yogo (2006) and Welch and Goyal (2008). In this section,we estimate our three predictive models to find evidence of predictabiity using macro variables such as T-bill rate and term spread as potential predictors for stock excess returns.

Estimation results are presented in Table 1. Panel A reports estimation results when the T-bill rate was used as a predictor for stock excess returns, while Panel B reports results when term spread was used as a predictor. In each panel, OLS estimation results of the PRM are reported in the first column. The next two columns show estimation results from the CRS model, while the last two columns present results from the ERS model. For two-state regime switching models, estimates for the regime switching parameters are presented side by side. The left $(s_t = 0)$ shows estimates in a low volatility regime, while the right $(s_t = 1)$ shows those in a high volatility regime.

The estimated volatility of stock excess returns in the state 1 $(\overline{\sigma_u})$ is almost three times bigger than that in the state 0 $(\underline{\sigma_u})$. The OLS estimates of the traditional predictive regression model show that the estimated β are not significant, which implies that none of macro variables seem to have any predictive power for stock excess returns. However, two-state regime switching models demonstrated predictability at least in the low volatility regime. It is noticeable that the predictive power was significantly observed at least when stock market is less volatile using two-state volatility regime switching models, though none of them seemed to have significant predictability using a simple predictive regression model.

Significant predictability in the low volatility regime is consistent with many previous papers which support predictability of macro variables. Fama and French (1989) and Chen (1991) emphasized a consumption smoothing motive to explain why economic growth forecasting variables could also play important role in predicting asset returns. A forecasting variable for the fundamental of an economy can also indirectly forecast asset returns because people will reduce their savings when future economic growth is expected, increasing returns on asset. Chen (1991) empirically showed that T-bill and term spread are valid indicators for future economic growth, implying that they might also have predictive power for stock returns. Campbell and Yogo (2006) and Welch and Goyal (2008) also indicated that T-bill rate and term spread have predictability for stock excess returns in their model.

However, regime switching models indicate that such predictability is only restricted to the period when the stock market is less volatile. When stock return is in high volatility regime, the estimation results in Table 1 imply that stock excess return is hardly predictable. This might be a result of an exogenous shock striking stock market which increases market volatility with rapidly changing asset prices. In such a case, the exogenous shock would be a main driving force in the asset pricing, making stock returns extremely hard to be predicted using any past information. Therefore, it might be hard to predict stock excess returns in the high volatility regime using T-bill or term spread, unlike in the low volatility regime.

Among the endogeneity parameters, ρ_2 was estimated to have quite substantial value while ρ_1 was not. This implies that the innovation of stock excess returns, especially the part that is uncorrelated to the predictor, affects the latent factor in the next period, determining the volatility regime of the following period. Since ρ_2 was significantly negative with a large magnitude, a negative shock on stock excess return seems to make high volatility regime more probable in the next period. This is consistent with the strong leverage effect. As reported at the bottom of Table 1, the null of no endogeneity $(\rho_1 = \rho_2 = 0)$ was rejected at a 5% significance level for both cases. The LR test results imply that the endogenous feedback effect should be considered, significantly increasing the explanatory power of the model.

3.2.2 Return Predictability with Common Price Ratios

The common predictive ratios has been considered as a potential predictor for stock returns in many previous literatures: Campbell and Shiller (1988), Lewellen (2004), Campbell and Yogo (2006), Welch and Goyal (2008), Cochrane (2008), and Choi *et al.* (2016). According to Campbell and Shiller (1988), asset returns rise when assets are underpriced relative to their fundamental values. The relationship between the fundamental value and price of an asset might allow fundamental to price ratio to predict stock returns.

As indicators for the price ratios, two variables were considered: the dividend-price ratio and the earning-price ratio. The estimation results are reported in Table 2. Panel A reports estimation results when the dividend-price ratio was used as a predictor for stock excess returns, while Panel B reports those when the earning-price ratio was used as a predictor. In both panels of Table 2, it is noticeable that estimated volatility of stock excess return in the state 1 $(\overline{\sigma_u})$ is almost three times bigger than that in the state 0 $(\underline{\sigma_u})$, as in Table 1. Therefore, stock excess return certainly seems to have switching volatilities. The traditional model, simply using least squares without considering regime switching property in stock return volatilities, implies that both price ratios have significant predictability for stock returns. However, once predictability is modeled separately for each volatility regime, predictability disappears in the high volatility regime; even in the low volatility regime for some cases.

When the dividend-price ratio was used as a predictor, Panel A shows that the predictability is significantly observed only in the low volatility regime. It seems that dividends over past year act as a good proxy for the fundamental value of stock, making the dividend-price ratio as a valid predictor for stock returns, at least in the low volatility regime. However, once the market becomes highly volatile, the predictability disappears. This might be due to the exogenous and unpredictable shocks on the stock market, as mentioned in the previous section. It is deducible that predicting stock excess returns using the dividend-price ratio, as well as T-bill rate and term spread, is extremely difficult in the high volatility regime because stock prices might move in unpredictable ways when the market is in the high volatility regime.

Panel B in Table 2 reports estimation results when the earning-price ratio was used as a predictor for stock excess returns. Unlike the estimation results from the traditional PRM, the
predictability of earning-price ratio was not significantly detected in any of volatility regimes when two-state volatility regime switching model was used. In addition to the earning-price ratio, smoothed real earnings-real price ratio (i.e., Shiller P/E ratio) in Campbell and Shiller (1988) was also tested as a predictor, but it did not show any significant predictability for stock returns, regardless of volatility regimes.

It is interesting that neither the earning-price ratio nor the smoothed real earnings-real price ratio have predictability for stock excess returns, even in the low volatility regime, which may be due to noisy earnings data. According to Fama and French (1988), the dividend-price ratio predicts returns better than the earning-price ratio since the latter is a noisier measure. They noted that higher variability of earnings makes the earning-price ratio a noisier measure than the dividend-price ratio, if it is unrelated to variations in expected returns.

One interesting point is that the earning-price ratio seems to have valid predictability when the simple PRM is used, while it loses its predictive power when volatility regime switching model is considered. This might have been an illusionary phenomenon caused by specific data characteristics that might distort standard estimation, such as persistence in predictor, correlations between returns and predictors and time-varying volatility in returns. For price ratios, π was significantly different from zero and ϕ was closely estimated to one. Moreover, the estimated values for regime switching parameter $\sigma_u(s_t)$ were highly different in each regime, implying that stock excess returns have switching volatilities. Under these conditions, as many papers have indicated, size distortion might occur, severely damaging the standard estimation using least squares in the PRM. By reflecting switching volatilities of stock returns, such problems might have been attenuated in the ERS model.

From Table 1 and 2, it was shown that ρ_2 was significantly estimated as largely negative value while ρ_1 was insignificant, regardless of what predictor for switching predictive equation was used. This implies that what affects the next period latent factor and volatility regime is the innovation of stock excess return, especially the part that is uncorrelated to the predictor. The null of no endogeneity $(\rho_1 = \rho_2 = 0)$ was strongly rejected as reported in the bottom of Table 2. The LR test results indicate that an endogenous feedback effect significantly increases the maximum log likelihood value, allowing the leverage effect to be considered within the model. While the predictability inference using the ERS model seems rarely different from that using the CRS model, the simulation results in Section 5 show that the regime process can be more sharply inferred via the ERS model yielding a gain in test power and bias.

3.2.3 Comparing the Estimated Predictability of Each Predictor

We also compare the testing results for stock return predictability using the PRM using OLS and the ERS models. For the ERS model, the joint null of no predictability $(i.e., \underline{\beta} = \overline{\beta} = 0)$ was tested using the likelihood ratio (LR) test. When the test results are reported, β is scaled by the estimated volatility ratio between the predictor and stock excess return (σ_v/σ_u) as in Campbell and Yogo (2006). Therefore, values of β as reported in Table 3 are $\beta \cdot (\sigma_v/\sigma_u)$ in an actual sense. This standardization enables us to compare predictability of different predictors easily. If β is estimated to be significant, we can say that an increase of one standard deviation of predictor (σ_v) would predict a $\beta \cdot (\sigma_v/\sigma_u)$ standard deviation change in expected stock excess returns.

When the T-bill rate was used as a predictor for stock excess returns, we failed to reject the null of no predictability using least squares. However, when the volatility regime was separately modeled in the ERS model, the joint null of no predictability under both regimes $(i.e., \underline{\beta} = \overline{\beta} = 0)$ was rejected significantly. When predictability in each volatility regime is tested respectively, test results

Table 3: Return Predictability Test Results

Predictor	OLS		ERS				
			$s_t = 0$		$s_t = 1$		p-value
	β	t-stat	$\underline{\beta}$	t-stat	$\overline{\beta}$	t-stat	for LR test
Treasury Bill Rate	-0.0067	-1.8977	-0.0121	-3.2154	-0.024	-1.5081	0.0012
Term Spread	0.0108	1.3292	0.0204	2.2483	0.006	0.2278	0.0738
Dividend-Price Ratio	0.0081	1.9587	0.0047	2.0815	0.0212	1.922	0.019
Earning-Price Ratio	0.0069	1.9472	0.0033	1.1334	-0.0067	-0.4422	0.4762

Notes: β coefficients obtained from OLS are scaled by $\widehat{\sigma_v}/\widehat{\sigma_u}$ while those obtained from ERS are scaled by $\widehat{\sigma_v}(s_t)/\widehat{\sigma_u}(s_t)$ in each volatility regime. The LR test was conducted with the null of no predictability $(\underline{\beta} = \overline{\beta} = 0)$.

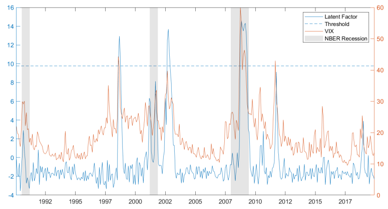
using the ERS model imply that T-bill rate significantly predict stock excess returns, at least in the low volatility regime. The term spread also seems to be an invalid predictor using least squares. However, it turned out that term spread also had significant predictability for stock excess returns, at least in the low volatility regime. Though the joint null was not rejected at the 5% level, the p-value (0.061) was reduced by a great amount compared to the value from OLS estimation. When the volatility regime is considered, macro variables seem to have valid predictive power at least in the low volatility regime.

When the dividend-price ratio was used as a predictor for stock excess returns, the null of no predictability was rejected at 5% level using least squares. The joint null of no predictability using the ERS model was also rejected. However, the ERS model additionally indicated that predictability was restricted only to the low volatility regime. More interestingly, the earning-price ratio could not reject the joint null of $\underline{\beta} = \overline{\beta} = 0$ at 5% level, though the null of $\beta = 0$ was rejected using OLS. Even when predictability under each volatility regime was tested respectively, both nulls $(i.e., H_0 : \underline{\beta} = 0$ and $H_0 : \overline{\beta} = 0)$ were failed to be rejected, implying that the earning-price ratio might not be able to predict stock excess return under any volatility regime.[6]

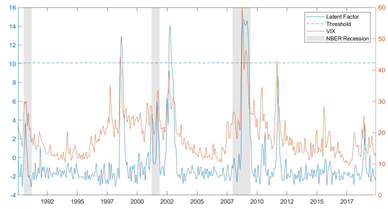
By comparing estimation results from the PRM and the ERS model, it was shown that ignoring volatility regimes might give significantly different infer-

[6]To show the robustness of our result, we performed the same analysis using 1926-2019 quarterly data, and 1947-2019 monthly data. When 1926-2019 quarterly data are used, we obtained the same result, except that the term spread no longer significantly predicts the excess returns. When 1947-2019 monthly data are used, only the T-till rate rejected the null of no predictability in low volatility regime.

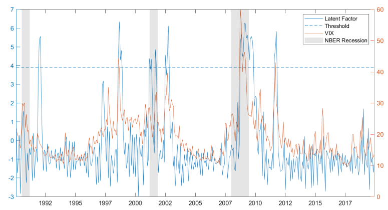
(a) T-bill rate as Predictor



(b) Term Spread as Predictor



(c) Dividend-Price Ratio as Predictor



(d) Earnings-Price Ratio as Predictor

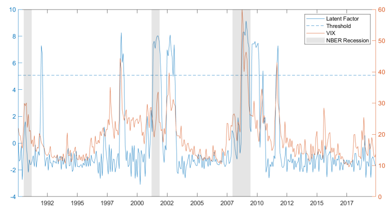


Figure 2: Extracted Latent Factors

ences on stock return predictability. When predictability exists only in the low volatility regime, it might be possible that predictability disappears when volatility regimes are not separately considered, due to the influence of high volatility regimes. The switching predictability with volatility regime might give explanation for the well-known instability in return predictability.

On the other hand, significantly detected predictive power under OLS might become insignificant after volatility regimes are separately considered. This might be the result of over-rejection problem caused by innovation correlation π that is significantly different from zero and highly persistent predictor series. In Section 5, we show that over-rejection problem could be alleviated when volatility regimes are properly considered.

The latent factor extracted from our model can be interpreted to have economic implications. In Figure 2, we compare the extracted latent factor with CBOE volatility index (VIX) and NBER recession periods. The extracted latent factor behaves similarly to VIX, and the high volatility regimes of our model mostly coincide with the NBER recession periods. From this result, we may conclude that our extracted latent factor well represents economic fundamentals.

4. SIMULATION

In this section, we conduct a Monte Carlo simulation to examine the effect of stock excess return volatility regimes, the persistence of predictor and innovation correlations between stock returns and predictors on three models: the PRM, the CRS model and the ERS model.

The first part of simulation was conducted to check if a regime-switching model is needed for β inference even when predictive power does not exist in both volatility regimes. Based the simulation results in Section 4.2, if volatility regimes are ignored as in the PRM, a test would yield serious distortions in terms of test size and bias, compared to the ERS model.

The second part of simulation considered time varying predictability with the volatility regime. Simulation results showed that the PRM cannot provide any valid inferences when predictive power exists only in the low volatility regime. In addition, the importance of the endogenous feedback effect is emphasized in Section 4.3, indicating that the ERS model is superior to the CRS model in terms of test power and finite sample bias. It seems that the ERS model can infer state process more sharply compared to the CRS model, allowing underlying time series to be reflected upon transition probability which yields power gain and bias improvement.

4.1. SIMULATION MODEL

The main simulation model we consider is given as

$$\begin{aligned}
y_t &= \beta(s_t)x_{t-1} + \sigma_u(s_t)u_t,\\
x_t &= \phi x_{t-1} + \sigma_v(s_t)v_t,\\
u_t &= \pi v_t + \sqrt{1-\pi^2}\varepsilon_t,
\end{aligned}$$

where $\beta(s_t) = \underline{\beta}(1-s_t) + \overline{\beta}s_t$ and $\sigma_u(s_t) = \sigma_v(s_t) = 0.03(1-s_t) + 0.10s_t$.

To allow the strong persistency in the predictor, a near unit root process is assumed as follows.

$$\phi = 1 - \frac{c}{n}$$

In our simulations, the two different sample sizes n were considered as 250 and

500. For each n, we considered the different values of c as $c = 0, 2, 10$.

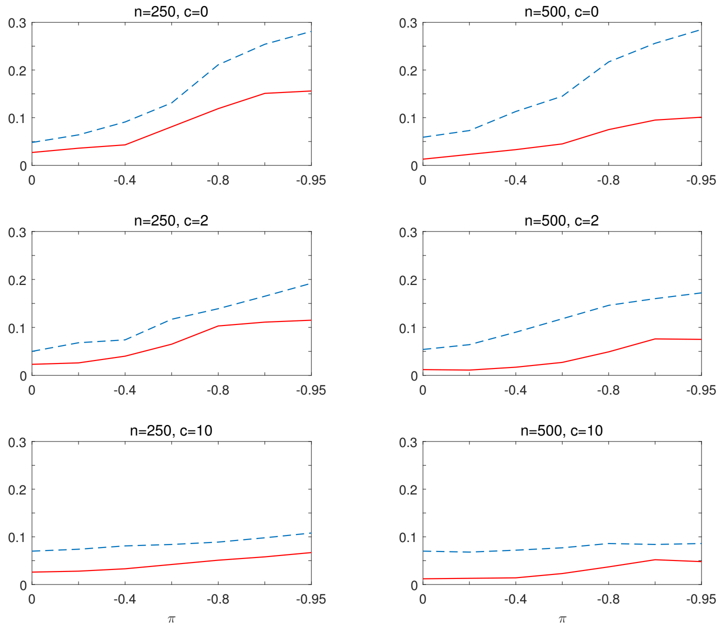
$$\begin{aligned} f_{t+1} &= \lambda f_t + \eta_{t+1}, \\ s_t &= 1\{f_t \geq \tau\}, \\ \begin{pmatrix} v_t \\ \varepsilon_t \\ \eta_{t+1} \end{pmatrix} &=_d N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & \rho_1 \\ 0 & 1 & \rho_2 \\ \rho_1 & \rho_2 & 1 \end{pmatrix}\right), \end{aligned}$$

where we set the autoregressive coefficient of the latent factor as $\lambda = 0.9$ and a threshold for each regime as $\tau = 0$. Two endogeneity parameters ρ_1 and ρ_2 were set as $\rho_1 = 0$ and $\rho_2 = -0.9$. To make make our simulation more realistic and practically more relevant, all the simulation parameters are approximated as closely as possible from our empirical results in the previous section. As previous literatures have been pointed out, persistence in predictor, correlations between returns and predictors might cause problems in test size and finite sample bias. Therefore, we investigate the testing size distortion y changeing the values of ϕ and π that measure the persistence and correlations, respectively.

4.2. VOLATILITY REGIMES

In this section, we first assume that predictability does not exist under any volatility regime $(i.e., \underline{\beta} = \overline{\beta} = 0)$. If this is so, one might argue that separately considering volatility regimes does nothing more than harm the parsimoniousness of the model. However, it was shown by simulation results that the ERS model could alleviate many problems in hypothesis testing and estimation, compared to the PRM. When the predictor is highly persistent and stock return innovation is correlated with that of the predictor, OLS in the PRM might yield size distortion with significant finite sample bias in β, whereas the ERS model can reduce size distortion and finite sample bias by separately modeling volatility regimes.

Figure 3 presents the finite rejection rate of the true null that there is no predictability. For the traditional model estimated using least squares, the null $H_0 : \beta = 0$ was tested against the alternative of $H_1 : \beta \neq 0$ with 5% significance level. For the ERS model, which considers volatility regimes with an endogenous feedback effect, the joint null $H_0 : \underline{\beta} = \overline{\beta} = 0$ was tested against the alternative that $H_1 : \underline{\beta} \neq 0$ or $\overline{\beta} \neq 0$ with 5% significance level. In Figure 3, simulation results are presented as figures for each (n, c) with π varying from 0 to -0.95 gradually. The finite rejection could be controlled by no more than 10% for all π values, especially when sample size is as large as 500. When the predictor was



Notes: The dashed blue line indicates the rejection rates for OLS with the null of $H_0 : \beta = 0$ and the solid red line is for the ERS model with the null of $H_0 : \underline{\beta} = \overline{\beta} = 0$

Figure 3: Finite Rejection Rate using OLS and ERS Model

not highly persistent $(c = 10)$, least squares estimation did not fail miserably, but still, the ERS model showed a smaller size distortion.

However, the test tended to be undersized when π was around zero. The predictability test using the ERS model seems conservative in a sense that it rejects too little under the true null when innovation correlation between stock returns and the predictor was not strong enough. Still, it is noticeable that test size using the ERS model was quite accurate when π was far from zero, which is important considering that the innovation correlation between stock excess returns and price ratios were estimated to be around -0.6 in Section 3.

Once volatility regimes are considered in the model, the bias in β can be reduced by almost half compared to OLS estimate. Stambaugh (1999) showed that the bias in β is proportional to the bias in ϕ and π. A slight modification of

Stambaugh (1999) yields

$$E[\widehat{\beta}(s_t) - \beta(s_t)] = \left(\frac{\sigma_u(s_t)}{\sigma_v(s_t)}\pi\right) E[\widehat{\phi} - \phi] \tag{9}$$

in our model. The derivation of 9 is skipped here. When the volatility regimes in stock excess returns and predictors are properly modeled, the bias in the autoregressive coefficient of the predictor (ϕ) can be reduced, making $\beta(s_t)$ less biased. The detailed results of β using OLS and those in $(\underline{\beta}, \overline{\beta})$ using the ERS model are skipped here to save space, but all n, c and π, the ERS model yields a smaller bias in β for each state $(\underline{\beta}, \overline{\beta})$ compared to the PRM estimated using least squares. Bias is reduced almost by half under each volatility regime using the ERS model. Overall, the ERS model can reduce size distortion and finite sample bias compared to the PRM when stock excess return and predictors have switching volatilities.

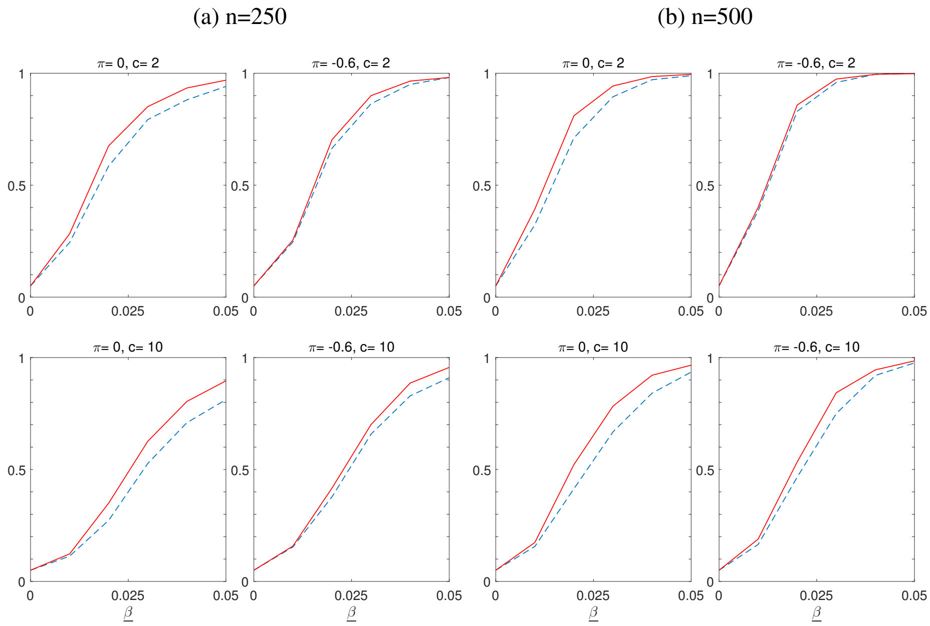
4.3. THE ENDOGENOUS AUTOREGRESSIVE LATENT FACTOR

In this Section, predictability was assumed to exist only in the low volatility regime $(i.e., \overline{\beta} = 0)$. If so, the traditional model, which ignores volatility regimes, cannot produce any valid inferences. Among two-state regime switching models, our simulation results show that the ERS model with the endogenous feedback effect of time series on the next period volatility regime, performs better than the CRS model in terms of test power and bias.

We assume that the predictability parameter $\beta(s_t)$ is $\underline{\beta}(1 - s_t) + 0 \cdot s_t$ with $\underline{\beta}$ ranging from zero to a non-zero value in fixed increments; from 0 to 0.05 with an increment of 0.1 when $n = 250$ and from 0 to 0.025 with an increment of 0.005 when $n = 500$. Since we focused on varying $\underline{\beta}$, we only considered two differenet values of π as $\pi = \{0, -0.6\}$ in this Section. Such π values are empirically relevant according to the estimation results in Section 3.

Figure 4 compares simulated powers of the CRS model and the ERS model in the low volatility regime. The test powers were all adjusted to have exact 5% size using the simulated critical values. Figure 4-(a) plots power functions when $n = 250$ while 4-(b) plots those when $n = 500$. For each panel, figures in the left column are for $\pi = 0$ while those in the right column are for $\pi = -0.6$. Compared to $\pi = -0.6$, power gain was generally bigger when $\pi = 0$. This is reasonable considering the correlation between u_t and η_{t+1} is $\pi\rho_1 + \sqrt{1-\pi^2}\rho_2$ and between v_t and η_{t+1} is ρ_1. In this simulation, ρ_1 was set to zero since it was estimated to have an insignificant value in our empirical results. This implies that only the part of stock excess return innovation, that is uncorrelated to the

Figure 4: Simulated Power Functions



Notes: Figure 4 presents simulated powers of the CRS model and the ERS model in low volatility regime. The dashed blue line is for the CRS model and the solid red line is for the ERS model. The test powers were all adjusted to have exact 5% size using the simulated critical values. Figure 4-(a) shows simulated power for n=250 and Figure 4-(b) shows simulated power for n=500.

predictor, affects the next period volatility regime. With $\rho_1 = 0$, innovation of underlying time series y_t (u_t) affect volatility regime of the next period due to its correlation with η_{t+1} by $\sqrt{1-\pi^2}\rho_2$. Therefore, compared to $\pi = -0.6$, $\pi = 0$ resulted in a stronger endogeneity effect on the state process. This might be the reason why power gain obtained from allowing an endogeneity effect is bigger when $\pi = 0$. All plots in Figure 4 indicate that the ERS model has power gain compared to the CRS model, for all cases of n, π and c.[7]

[7] In addition to power gain, the ERS model can also reduce finite sample bias in $\beta(s_t)$ compared to the conventional regime switching model. The detailed results are skipped due to the limited space. We also can show that the ERS model can distinguish regimes more sharply than CRS during transition periods, using additional information obtainable from the previous period

5. CONCLUSION

The stock return might behave differently when the stock market is relatively stable and highly volatile, making return predictability vary with its volatility regimes. This paper introduces a two-state regime switching model with an endogenous feedback effect, which allows one to separately examine stock return predictability in low and high volatility regimes. According to empirical analysis using the ERS model, it was clearly shown that none of the tested predictors can significantly predict stock excess returns under the high volatility regime. Only in the low volatility regime did the dividend-price ratio and macro variables such as T-bill rate and term spread show significant predictive power for stock excess returns. The earning-price ratio turned out to be the insignificant predictor even in the low volatility regime.

The ERS model was more realistic in the context of the return predictability test, compared to the conventional Markov switching (CRS i.e., conventional regime switching) model. It was expected that the stock return innovation would affect the next period volatility regime. A negatively estimated endogeneity parameter ρ_2 was consistent with the leverage effect, indicating that a negative shock on current returns tends to increase the next period volatility. On the other hand, ρ_1 was insignificant no matter what predictor for stock return prediction was used. These results implied the existence of the endogenous feedback effect of the underlying time series on regime process; but it is not past value of predictor series what affect the next period volatility regime. Rather, it is past value of return innovation especially the part that is uncorrelated with predictor series.

The ERS model could also alleviate the problems in the estimation and hypothesis testing. Compared to the traditional model estimated using least squares, the ERS model could relieve finite sample bias and the over-rejection problem. The endogenous feedback effect channel modeling, as proposed by Chang *et al.* (2017), enabled a gain in test power and bias improvement compared to the conventional model. Through endogenous feedback effect channel, the additional information from the underlying time series could be reflected in the latent factor and state process, resulting in sharper inference of the state process particularly during the transition periods. This might have contributed to benefits in hypothesis testing and estimation.

As can be seen from Section 4, the dividend-price ratio and macro variables have valid predictability only in the low volatility regime; and no predictability significantly observed in the high volatility regime. Such switching predictabil-

underlying time series. The details are also skipped here.

ity might be the plausible explanation for widely-observed instability of return prediction in the related literatures. Though return predictability does exist when the stock market is less volatile as the ERS model demonstrated, there remains a question whether it can provide a practical help for real investors, due to the limitation that the state prediction can hardly be perfect. Although the ERS model achieved the improved state inference compared to the conventional model, it has around 93 percent accuracy (not a hundred percent). The future research might examine how useful it is to implement the return predictability limited to the low volatility regime, using the prediction on future volatility regime on which return predictability depends on.

To sum up, the contribution of the ERS model is to suggest a method to respectively test return predictability for different volatility regimes with improved state inference. An endogenous feedback effect channel allowed the additional information from the underlying time series to be reflected in the transition probability, resulting in much sharper state inference and, thereby, better estimation results. Nevertheless, it is the limitation of the ERS model that the problems caused by prediction persistence and innovation correlation have only been attenuated, not solved. It would be of great use if the data characteristics mentioned above could be comprehensively managed within a volatility regime switching model, resulting in a complete solution for over-rejection and the bias problem without the model losing its ability to examine return predictability separately for different volatility regimes.

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