

## The ‘Pay for Luck’ Puzzle: A Macroeconomist’s View\*

Hyung Seok E. Kim<sup>†</sup>

**Abstract** The present study considers a model of delegated management where risk-averse shareholders delegate their firm’s management to self-interested executives/managers, but within the general equilibrium context of Pigouvian cycles. A socially optimal class of managers’ remuneration contracts is identified in this Pigouvian environment where business fluctuations could be driven by private-sector expectations that are unrelated to economic fundamentals. These general equilibrium considerations have two primary implications. First, the “pay-for-luck” phenomenon, largely emphasized in the executive compensation literature, arises as an aggregate equilibrium outcome, thereby providing a corollary resolution of the corresponding “pay-for-luck” puzzle. While rendering the CEO-to-worker pay ratio essentially irrelevant to social welfare, the delegated management economy with the manager’s first-best compensation contracts may produce economy-wide welfare losses more than one order of magnitude larger than the Lucasian cost-of-business-cycle estimate, which constitutes a second point.

**Keywords** Stochastic growth model; delegated management; executive compensation; optimal contract; Pigouvian cycles

**JEL Classification** E32, E44

---

\*I would like to thank two anonymous referees, and the Editor, Hye Mi You, for comments that are subsumed throughout the paper.

<sup>†</sup>School of Science and Technology Policy, KAIST, E-mail: hyung.seok.eric.kim@gmail.com

## 1. INTRODUCTION

The present study constructs a model of delegated management where risk-averse shareholders delegate their firms' management to self-interested and risk-averse executives/managers, but in a macroeconomic Pigouvian-cycle setting. Within the context of the "separation of ownership and control" in the modern US coportation culture, some general equilibrium implications of the manager's remuneration contracts are deduced: these contracts are required to be aligned with the twin goals of the firm owner's interest and the firm's maximized value. Along the lines of the latter requirement, this paper is particularly interested in the problem of designing socially optimal executive incentive compensation contracts situated in a macroeconomic environment of Pigouvian business fluctuations.

The key to optimal contracting in this macroeconomic context, I argue, must be the appropriate "balance" between the desired contract's incentive component, propotional to a non-tradable equity position, and its salary counterpart indexed to the aggregate wage bill.<sup>1</sup> Conforming to the standard "moral hazard" literature, it is not surprising that the optimal contract of interest takes the form of linearity, suggesting that the manager's compensation must be linearly proportional to her performance measure, suitably chosen even in a dynamic general equilibrium setting. The economy's intertemporal decision-making, however, places an additional restriction on the optimal contractual form, requiring that the manager's stochastic discount factor must be aligned with the shareholder's counterparts to generate the firm's future dividends. The latter requirement suggests that the optimal contract's performance measure will be strongly tied to the aggregate state of the economy, standing 180 degrees to what standard incentive theory offered. The benchmark story of moral hazard, by contrast, tells us that the aggregate state of the business cycle will be irrelevant to determine the manager's compensation when her performance measure is statistically linked with her hidden efforts in a *monotone likelihood ratio property* (MLRP) manner, as the contract's performance criterion will be a sufficient statistic for any bilateral resoultions of this moral hazard problem.

The present managerial contract may therefore bring a natural resolution to the "pay-for-luck" puzzle, one of conspicuous anomalies in the executive compensation literature: the "pay-for-luck" puzzle indicates that a CEO's pay seems more closely linked with a broad-based market index such as the market portfo-

---

<sup>1</sup>At the opposite extreme, the contract's huge imbalance between the prior two, with the former substantially emphasized, will ultimately lead to an economy of concentrated equity ownership. See Section 3 for a further discussion of the latter economy.

lio return, typically representative of the state of the business cycle, rather than her own firm's stock return.<sup>2</sup> It must be noted, moreover, that the present dynamic economy is largely vulnerable to Pigouvian cycles, what the literature often refers to as news-driven business cycles, thereby making the case that the “pay-for-luck” phenomenon linked with the manager's socially optimal compensation contracts should be irrelevant to any distortions of social welfare, no matter how large the economy's aggregate fluctuations would be even in response to her own expectations that are unrelated to economic fundamentals.

As a corollary argument, the presence of Pigouvian fluctuations offers a litmus test for the managerial contract's economy-wide welfare costs in the Lucasian sense. While inducing managers to adopt the first-best investment and hiring decisions from a social perspective, these socially designed contracts may lead to economy-wide welfare losses more than one order of magnitude larger than Lucas (1987, 2003) cost-of-business-cycle estimate. More interestingly, the cost of business-cycle-frequency fluctuations turns out to be irrelevant to the CEO-to-worker pay ratio suitably parametrized in the present context, despite being sensitive to an increase in the economy's overall risk aversion—the latter observation is completely in line with the basic conclusion of the cost-of-business-cycle literature.

The suggestion in this paper that the “pay-for-luck” puzzle may be addressed by first-best executive compensation contracts within the context of a macroeconomic business-cycle model is not new. Notable references are Danthine and Donaldson (2015) and Donaldson and Kim (2023) to mention but a few.<sup>3</sup> The

---

<sup>2</sup>Bertrand and Mullainathan (2003) first identified the “pay-for-luck” puzzle, which empirically proves to be a positive relationship between CEO compensation and oil price shocks. Garvey and Milbourn (2006) subsequently also find that the puzzle is isomorphic to the executive pay's unambiguously strong link with market returns.

<sup>3</sup>The latter further construct an otherwise standard real business cycle model, where the twin contractual forms of linearity and “option-like” convexity vis-à-vis managers' performance measures, whether first-best or sub-optimal, are introduced. This paper also explores the cost of Prescottian business fluctuations under various contractual scenarios, suggesting that only the latter contractual type associated with Pareto sub-optimality will produce large cost-of-business-cycle estimates. Along the line of Pareto inefficiency, Kim (2023) introduces a model of delegated management, wherein the one-to-one proportionality between a “CEO external managing premium” and an *investment wedge* in the spirit of Chari *et al.* (2007) can be rationalized through Nash-bargained contracts of executive compensation: the latter, resulting from the manager's under- or over-investment decisions due to the former's *raison d'être*, is seen to be a main driver behind the economy's Pareto inefficiency. The model in Kim (2023) accordingly posits that the soaring ratio of CEO pay to that of the average worker, characteristic of the modern US economy, should be viewed as a bellwether for the economy's significant Pareto suboptimality, in sharp contrast to the present thesis.

key distinction between the present model and theirs, however, is that Pigouvian shocks are a primary driving force behind the former, while the latter’s business cycles are propagated in response to benchmark *Prescottian* supply shocks. Accordingly, the present model creates a larger Lucasian cost of fluctuations than those Prescottian counterparts, despite the model’s Pareto-efficient business fluctuations.

The paper proceeds as follows. Section 2 presents the basic model and its implied Pareto-optimal managerial contract, culminating in Theorem 1. Section 3 introduces Pigouvian shocks to the delegated management economy, and explores the economy’s Pigouvian-cycle consequences, particularly, vis-à-vis the cost of fluctuations. Section 4 concludes the paper.

## 2. THE MODEL

I begin by describing a public-information equilibrium, abstracting from both moral hazard and adverse selection considerations, although the case of the manager’s hidden efforts within the context of moral hazard will be discussed later (See Theorem 1). The model I consider is essentially a discrete-time infinite horizon economy with two distinct infinitely-lived agent types, “managers” and “shareholder-worker-consumers,” with the latter type often referred to as “shareholders” for economy of presentations. These groups are uniformly distributed, respectively, on sets of positive Lebesgue measure,  $\mu$  and a unit interval. Self-interested managers are selected at the beginning of time to manage a single stand-in firm permanently and are accordingly assumed to make all the relevant decisions in view of maximizing their own intertemporal utility. Shareholders thus seek a contract that incentivizes the manager to select investment and hiring policies that are maximizing shareholders’ utility.

The representative shareholder-worker–consumer is confronted with a work versus leisure decision and a portfolio investment decision. The form of his optimization problem is standard. The representative shareholder’s problem reads:

$$V^s(\Omega_0^s) = \max_{\{n_t^s, c_t^s, z_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u^s(c_t^s - bc_{t-1}^s) - H(n_t^s)] \quad (1)$$

s.t.

$$c_t^s + p_t^s s_{t+1} \leq (p_t^s + d_t) s_t + w_t n_t^s$$

given his information set  $\Omega_t^s = \{s_t, p_t^s, d_t, w_t; c_{t-1}^s\}$ .

In problem (1),  $u^s(\cdot)$  is the shareholder's period utility of consumption,  $H(\cdot)$  his disutility of work function,  $c_t^s$  his period  $t$  consumption,  $n_t^s$  his period  $t$  labor supply,  $s_t$  the fraction of the firm's single equity share he holds at the start of period  $t$ , and  $p_t^s$  the equity price of the firm. The parameter  $b$  represents the agent's habit formation parameter — I assume both agent types have habit preferences to ensure the existence of Pigouvian cycles. I also assume both agents have the same discount factor.

The necessary and sufficient condition problem (1) are characterized by

$$u_c(c_t^s - bc_{t-1}^s) - b\beta\mathbb{E}(u_c(c_{t+1}^s - bc_t^s) | \Omega_t) = \lambda_t^s \quad (2)$$

$$\lambda_t^s w_t = H_n(n_t^s) \quad (3)$$

$$p_t^s = \mathbb{E}_t\left[\frac{\lambda_{t+1}^s}{\lambda_t^s}(p_{t+1}^s + d_{t+1})\right] \quad (4)$$

There is a single 'stand-in' firm which provides output,  $\{y_t\}$ , each period. Its production technology is of the Cobb-Douglas form,

$$y_t = f(a_t, v_t k_t, n_t) = a_t (v_t k_t)^\alpha n_t^{1-\alpha} \quad (5)$$

where  $a_t$ ,  $v_t$ ,  $k_t$ , and  $n_t$  denote, respectively, the level of Hicks-neutral technology, the rate of capital utilization, the stock of capital, and aggregate hours supplied in period  $t$ . The evolution through time of the firm's capital stock is given by

$$k_{t+1} = \left[1 - \phi\left(\frac{i_t}{i_{t-1}}\right)\right] i_t + [1 - \delta(v_t)]k_t \quad (6)$$

where  $i_t$  denotes aggregate investment in period  $t$ ; in equation (6),  $\phi$  is an adjustment cost function that satisfies the twin properties of  $\phi(1) = \phi'(1) = 0$ , and  $\phi''(1) > 0$ .<sup>4</sup> Directly adapted from Jaimovich and Rebelo (2009), the rate of capital depreciation as a function of  $v_t$ ,  $\delta(v_t)$ , is formulated by its convexity with respect to  $v_t$  (i.e.,  $\delta'(v_t) > 0$  and  $\delta''(v_t) \geq 0$ ). The economy's labor market is competitive with all firms hiring at the going wage rate  $w_t$ . The product of  $v_t$  and  $k_t$ , the measure of capital services, will be seen later to affect a shift in labor demand in response to Pigouvian (news) shocks.

At the beginning of period  $t$ , the firm's manager observes the productivity parameter  $a_t$ ; she then undertakes her utility-maximizing decisions  $(i_t, n_t)$  in light of her remuneration contract  $\mathbb{C}^m$ . The manager is not given access to capital markets and thus consumes her income.<sup>5</sup> Letting  $c_t^m$  denote the manager's period  $t$

<sup>4</sup>This specification guarantees that the model's steady state equilibrium does not depend on the adjustment cost parameter,  $\kappa \equiv \phi''(1)$  (Christiano *et al.*, 2005).

<sup>5</sup> Under the optimal contract, security trading is redundant for the manager.

consumption, thus, one can conclude that  $c_t^m = \mathbb{C}^m \cdot n_t^m$  with her labor effort,  $n_t^m$ . It should come as no surprise to presume that the manager is banned from trading the equity issued by the firm she manages: not only does this rule protect shareholders from insider trading, but it also prevents the manager from using financial markets to insure against her own managerial risk solely derived from the contract's certain performance measures.

In line with specification (5), take as a benchmark the scenario where the manager's effort as a factor of production is inessential in the production process, yet where the manager is offered a performance-based contract, so that the manager's real economic decisions amount to investment and hiring ones in equilibrium. Following Danthine and Donaldson (2015) and Donaldson and Kim (2023), the firm's free cash flow before payments to managers,  $\hat{d}_t$ , will be viewed as a plausible sufficient statistic for the manager's performance measure ensuring Pareto optimality;  $\hat{d}_t$ , satisfying  $\hat{d}_t = f(a_t, v_t k_t, n_t) - n_t w_t - i_t$ , will prove to be the case, indeed.

Letting  $u^m(\cdot)$  represent the manager's utility-of-consumption function, and  $\beta$  her subjective discount factor, the manager's problem reads:

$$\begin{aligned}
 V^m(\Omega_0^m) &= \max_{\{n_t^s, i_t, v_t, c_t^m, n_t^m\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u^m(c_t^m - b c_{t-1}^m) & (7) \\
 &\text{s.t.} \\
 c_t^m &= \mathbb{C}^m(d_t; n_t^s, i_t, v_t; \Omega_t^m) n_t^m \\
 d_t &= f(a_t, v_t k_t(f), n_t(f)) - n_t w_t - i_t - \mu c_t^m \equiv \hat{d}_t - \mu c_t^m \\
 k_{t+1} &= \left[ 1 - \phi \left( \frac{i_t}{i_{t-1}} \right) \right] i_t + [1 - \delta(v_t)] k_t, \\
 n_t^m &\leq 1
 \end{aligned}$$

where the manager maximizes her consumption streams in a "self-interested" manner given her information set  $\Omega_t^m = \{k_t, i_{t-1}, z_t; w_t; \mathbb{C}_t^m(\cdot); c_{t-1}^m\}$ .

The entire economy's total output can be distributed to meet the respective demands of consumption and investment goods:

$$y_t = c_t^s + \mu c_t^m + \frac{i_t}{z_t}, \quad (8)$$

where  $\{\tilde{z}_t\}$  represents the level of investment-specific technology as in Greenwood *et al.* (2000) and Fisher (2006). In formulation (8), we do not rule out the possibility that both exogenous technology shocks,  $\{\tilde{a}_t\}$  and  $\{\tilde{z}_t\}$ , have stochas-

tic trends, although only the latter case will be considered in the model's Pigouvian-cycle experiment.

The economy's decentralized equilibrium can then be formalized as follows:

**Definition 1.** *Equilibrium in the delgated management economy consists of a set of decision rules*

$\{c_t^s(\cdot), c_t^m(\cdot); n_t^s(\cdot), n_t^m(\cdot); s_{t+1}(\cdot); i_t(\cdot), n_t(\cdot); v_t(\cdot)\}$  and a set of wage contracts and price functions  $\{w_t(\cdot); p_t^s(\cdot), d_t(\cdot)\}$  given the information set of aggregate states  $\Omega = \{k_t; a_t, z_t\}$  and the managerial contract  $\mathbb{C}_t^m(\cdot)$  such that

(i)  $\{c_t^s(\cdot), n_t^s(\cdot); s_{t+1}(\cdot)\}$  satisfies the shareholder-worker-investor's first order conditions given the information set  $\Omega_t^s = \{s_t, p_t^s, d_t, w_t; c_{t-1}^s\}$

(ii)  $\{c_t^m(\cdot), n_t^m(\cdot); i_t(\cdot), n_t^s(\cdot), v_t(\cdot)\}$  satisfies the delegated manager's first order conditons given the information set  $\Omega_t^m = \{k_t, i_{t-1}; a_t, z_t; w_t; \mathbb{C}_t^m(\cdot); c_{t-1}^m\}$

(iii)  $w_t(\cdot)$  is the market-clearing wage in the labor market such that  $n_t = n_t^s$

(iv)  $p_t^s(\cdot)$  is the market clearing price in the equity market such that  $s_t = 1$

(v)  $\{p_t^s(\cdot), d_t(\cdot)\}$  satisfies the Lucas' asset pricing (vi) The economy follows the law of motion:  $k_{t+1} = [1 - \delta(v_t)]k_t + i_t[1 - \phi(\frac{i_t}{i_{t-1}})]$

(vii) the resouce constraint is binding:  $c_t^s + \mu c_t^m + \frac{i_t}{z_t} = y_t$ .

As a counterpart to formulation (1) described above, a central planner's problem is now introduced in the spirit of the second welfare theorem:

**Definition 2.** *Given the aggregate state of the economy,  $\Omega_t = \{c_{t-1}^s, c_{t-1}^m, i_{t-1}, k_t, z_t\}$ , the central planner's problem is as follows:*

$$V^{CP}(\Omega_0) = \max_{\{c_t^s, c_t^m; n_t^s; v_t; i_t\}} \mathbb{E}_0 \sum \beta^t [\theta \mu u(c_t^m - bc_{t-1}^m) + u(c_t^s - bc_{t-1}^s) - H(n_t^s)] \quad (9)$$

s.t.

$$y_t = c_t^s + \mu c_t^m + \frac{i_t}{z_t}$$

$$k_{t+1} = [1 - \delta(v_t)]k_t + i_t[1 - \phi(\frac{i_t}{i_{t-1}})]$$

$$y_t = f(a_t, v_t k_t, n_t^s)$$

$$a_t \equiv \text{Hicks-neutral technology}$$

$$z_t \equiv \text{investment-specific technical change}$$

where  $\theta$  is an arbitrary relative welfare weight attributed to an individual manager when the welfare weight of an individual shareholder is normalized to 1 and  $v_t$  is the rate of capital utilization.

The following theorem makes clear how the delegated management and Pareto-optimal economies coincide through the optimal contract of form (10).

Under formulation (10), it is further shown that introduction of the manager's unobservable efforts under the heading "private information" does not change the delegated management economy's Pareto optimality.

**Theorem 1.** *Consider a manager's compensation contract of the form*

$$\mathbb{C}^m = \mathbb{C}^m(\widehat{d}_t, w_t n_t; \mathbf{s}_t) = A + \varphi \left[ w_t n_t + \widehat{d}_t \right]^\eta, \quad (10)$$

where  $\mathbf{s}_t$  represents the aggregate state vector of the economy in a recursive-equilibrium sense.

- a. *If  $u^s(\cdot) = u^m(\cdot)$ , then a contract of form (10), with  $A_t \equiv A = 0$ ,  $\varphi > 0$  and  $\theta = 1$ , is optimal in the sense that a manager subject to this contract will select investment and hiring plans that lead to the first-best equilibrium for the model economy that coincides with the planner's solution to formulation (2).*
- b. *Consider the case where the manager's unobservable effort,  $e_t$ , is essential to production as identified by the production function*

$$f(a_t, v_t k_t, n_t, \mu e_t).$$

*Conforming to the moral hazard literature, it is assumed that the manager's effort,  $e_t$ , is private information, uncertain from the perspective of the consumer-worker-investor, and distinct from the (observable) aggregate productivity uncertainty  $\{\tilde{a}_t\}$ . Further impose the statistical assumption of the monotone likelihood ratio property (MLRP), i.e., the intuitive requirement that a high-performance realization or high-output state reflects favorably on the manager's choice of effort, thereby formalizing the following condition:*

$$\frac{d}{dy} \left[ \frac{\partial}{\partial e} \log \mathbb{P}(y, a|e) \right] \geq 0$$

*where  $\mathbb{P}(y, a|e)$  is the joint stationary probability density with respect to output  $y$  and the productivity shock  $a$  conditional on the chosen effort level. If the manager's first-best managerial contract as in a. is guaranteed in aggregate equilibrium, then her private information is irrelevant to the first-best allocation of resources.*

*Proof.* See Technical Appendix. For the case of moral hazard, see Donaldson and Kim (2023). The key to proving the latter part is Prescott and Townsend (1984) securitization scheme vis-à-vis private information.  $\square$



### 3. RESULTS

**Pigouvian Shocks** To introduce Pigouvian cycles into the present environment, I adopt the “anticipated shock” methodology of Donaldson *et al.* (2022), a revision of Schmitt-Grohé and Uribe, (2012) methodology.<sup>6</sup> I first reformulate the twin technological changes of  $\{\tilde{a}_t\}$  and  $\{\tilde{z}_t\}$ , respectively, in terms of growth-rate shocks:

$$\phi_t^a \equiv \frac{a_t}{a_{t-1}} \quad \text{and} \quad \phi_t^z \equiv \frac{z_t}{z_{t-1}}. \quad (11)$$

Assume further that permissible growth-rate shocks  $\{\phi_t^\Delta\}$ , where  $\Delta \in \{a, z\}$ , evolve over time according to the standard autoregressive form:

$$\log(\tilde{\phi}_t^\Delta / \bar{\phi}^\Delta) = \rho_\Delta \log(\phi_{t-1}^\Delta / \bar{\phi}^\Delta) + \varepsilon_{\Delta, t-j}^j. \quad (12)$$

In formulation (12),  $\rho_\Delta$  and  $\bar{\phi}^\Delta$  denote, respectively, the autoregressive coefficient of process  $\Delta$  and the process’s non-stochastic average rate of growth;  $\{\tilde{\varepsilon}_{\Delta, t-j}^j\}$  follows an i.i.d. Gaussian (normal) distribution with mean zero and standard deviation  $\sigma_\Delta$ .

The error term  $\varepsilon_{\Delta, t-j}^j$  represents an “anticipated” innovation to  $\phi_t^\Delta$ , observed by rational economic agents  $j$  periods earlier for  $j = 2, 4$  or  $8$ , but materializing later in period  $t$ . We attach the meaning of “news shocks” to  $\varepsilon_{\Delta, t-j}^j$  in the sense that forward-looking rational agents will react to anticipated news about Hicks-neutral and investment-specific technical changes in period  $(t-j)$ , although such period  $(t-j)$  news announcements will be unambiguously reified in period  $t$ .<sup>7</sup>

The following proposition guarantees that the present delegated management economy is susceptible to Pigouvian cycles, i.e., business fluctuations that could be driven by private sector expectations that are unrelated to the economy’s fundamentals, thereby rendering the ‘pay-for-luck’ puzzle more puzzling at the micro level, but offering the puzzle’s potential resolution at the aggregate level.

**Proposition 1.** *Consider the limiting case where  $\mu = 0$  and the representative consumer-worker’s labor supply is fixed. The delegated management economy*

<sup>6</sup>The key distinction between Donaldson *et al.* (2022) and Schmitt-Grohé and Uribe, (2012) methodologies is that the former employs the method of third-order perturbation for the welfare-cost reason, while the latter adopts the first-order counterpart for estimation reasons.

<sup>7</sup>Under formulation (12), these “news shocks” manifest themselves as *autonomous yet non-falsifiable* changes in the agent’s growth expectations constrained by her own anticipation horizon of  $j$  quarters when  $j = 2, 4$  and  $8$ . By contrast, the case of  $j = 0$  (i.e.,  $\varepsilon_{\Delta, t}^0$ ) represents contemporaneous, unanticipated shocks, whose significance is largely emphasized in the conventional RBC literature under the banner of the “Prescottian channel of supply shocks” (Prescott, 1986).

(1) displays Pigouvian cycles (impulse responses) in the sense of Jaimovich and Rebelo (2009) under the data-generating process (12).

*Proof.* By Theorem 1, it will suffice to check whether the planner's economy 2 can display Pigouvian cycles or not. Under the twin assumptions of  $\mu = 0$  and the representative consumer-worker's fixed labor supply, the central planning problem here is isomorphic to Christiano *et al.* (2010) Pigouvian-cycle economy.  $\square$

**Remark 2.** *Jaimovich and Rebelo (2009) suggest three necessary and sufficient conditions for the replication of Pigouvian cycles: the gradual capital adjustment of type (6), the variable capital utilization rate  $v_t$ , and a preference specification that generates a weak wealth effect on variable labor supply.*

**Cost of Fluctuations** Under the delegated management economy, how large are the costs of business fluctuations in the Lucasian sense? This question also relates to the broader one of how harmful the soaring ratio of CEO pay to that of the average worker, characteristic of the present-day US economy, would be from the standpoint of social welfare. In the present context, the latter question will be particularly appropriate and legitimate, as the arbitrary parameter  $\varphi$  in Theorem 1 can be construed as the ratio of CEO pay to that of the average worker. If any large number of  $\varphi$  had a tendency to produce the high cost of fluctuations, then any social justice, as commonsensical as it appears, would be immediately called for. The model's implication, however, flies in the face of "common sense," as summarized below.

**Corollary 1.** *Suppose that the manager's compensation contract of form (10) satisfies the Pareto-optimal contract requirements specified in Theorem 1. Under the limiting case of  $\mu = 0$ , the parameter  $\varphi$ , parameterized to mirror the ratio of CEO pay to that of the average worker, is irrelevant to the determination of the economy's real allocation and social welfare.*

Despite this irrelevance result, Proposition 1 facilitates the calculation of Lucas (1987, 2003) cost-of-business-cycle estimates in the Pigouvian-cycle-driven delegated management economy. Consider the planner's economy in formulation 2, if necessary, in its suitably detrended long-run equilibrium stochastic steady state. Let  $dM(\tilde{\mathbf{s}}_t)$  denote the joint ergodic probability distribution on the economy's (detrended) state vector  $\tilde{\mathbf{s}}_t$  and let  $\{c(\tilde{\mathbf{s}}_t)\}$  and  $\{n(\tilde{\mathbf{s}}_t)\}$  denote the corresponding (representative) consumer's ergodic (detrended) consumption and labor supply distributions resulting from the planner's efficient decisions.

Following Lucas (1987, 2003), there exists a constant  $\mathcal{K}$  such that

$$V^s((1 + \mathcal{K})c(\tilde{\mathbf{s}}_t), n(\tilde{\mathbf{s}}_t)) = V^s(c^{ss}, n^{ss}) \quad (13)$$

where  $V^s(\cdot)$  denotes the representative consumer's welfare function, respectively, in the case of her detrended *stochastic steady state* consumption stream  $(1 + \mathcal{K})c(\tilde{\mathbf{s}}_t)$  and unchanged labor supply  $n(\tilde{\mathbf{s}}_t)$  (LHS of (13)). The RHS of (13) gives her welfare in the case of her corresponding (deterministic) steady state consumption stream  $c^{ss}$  and steady state labor supply  $n^{ss}$ . Lucas (1987, 2003) then chooses to measure the cost of business cycles, as a share of steady state output, by

$$\Lambda = \frac{\mathcal{K} \int c(\tilde{\mathbf{s}}_t) dM(\tilde{\mathbf{s}}_t)}{y^{ss}}. \quad (14)$$

Table 1: Parameterization

	Parameter	Value	Attribution
$\alpha$	(capital income share)	$\alpha = .36$	Cooley and Prescott (1995); commonplace
$\alpha$	(capital income share)	$\alpha = .36$	Cooley and Prescott (1995); commonplace
$\delta(\bar{v})$	(quarterly capital depreciation rate)	$\delta(\bar{v}) = .025$	Kydland and Prescott (1982); Kaltenbrunner and Lochstoer (2010)
$\gamma^s$	(the representative consumer's coefficient of relative risk aversion)	$\gamma^s = 2$	Free parameter
$\gamma^m$	(the manager's coefficient of relative risk aversion)	$\gamma^m = \gamma^s$	Theorem 1
$b^s$	(the representative consumer's habit formation parameter)	$b^s = .9$	Christiano <i>et al.</i> (2010)
$b^m$	(the manager's habit formation parameter)	$b^m = b^s$	Theorem 1
$\beta$	(economy-wide quarterly subjective discount factor)	$\beta = .99$	commonplace; yields an annualized average capital return of 4%
$\bar{g}_a$	(the expected growth rate of neutral technology)	$\bar{g}_a = 1.0022$	Donaldson <i>et al.</i> (2022)
$\bar{g}_z$	(the expected growth rate of investment-specific technical change)	$\bar{g}_z = 1.0047$	Donaldson <i>et al.</i> (2022)
$\sigma_{\varepsilon,z}$	(S.D. of Investment-specific shocks)	$\sigma_{\varepsilon,z} = 0.60\%$	Donaldson <i>et al.</i> (2022)

(i) When factor markets are competitive the parameter  $\alpha$  is typically calibrated to reproduce the observed share of US capital income in total value added.

(ii) Mehra and Prescott (1985) disarmingly argue that  $(0, 10)$  is a reasonable range of possible values.

For the purpose of correct welfare evaluations, the third-order perturbation method is employed to solve numerically for the present model.<sup>8</sup> The key parameters underlying the “welfare-cost” assessments are summarized in Table 1.

Table 2 presents the model's main results. For comparison reasons, additional scenarios are also presented along with the benchmark case. The DD entry derives from Danthine and Donaldson (2015) model of delegated management, essentially isomorphic to the classic business-cycle model of Hansen (1985). The

<sup>8</sup>A primary source for this method can be found in Andreasen (2012).

RBC-Fisher entry, by contrast, refers to Fisher (2006) model of non-monetary business cycles with the twin stochastic trends of Hicks-neutral technology and investment-specific technical change. The phrase “Concentrated Equity Ownership” presents a model of limited asset market participation, essentially derived from Guvenen (2009), serving as the opposite extreme of the benchmark model where the managerial contract’s equity position is extremely magnified: the latter observation illustrates a huge imbalance between the contract’s incentive and salary components, with the former dramatically emphasized for incentive reasons, standing 180 degrees to the first-best contract of executive compensation.

MODEL <sup>(i)</sup>			
Benchmark <sup>(ii)</sup>	DD <sup>(iii)</sup>	RBC-Fisher	Concentrated Equity Ownership
−.1315% <sup>(iv)</sup>	−.0084%	−.0046% <sup>(v)</sup>	−.0600% <sup>(vi)</sup>

Table 2: Welfare Gains (Losses)

(i) Across the board, the representative agent’s risk aversion is equal to 2, except for the case of “Concentrated Equity Ownership” wherein agent heterogeneity in risk aversion is assumed. In the latter model, the manager-shareholder’s risk aversion parameter is equal to 2, while the non-shareholder-worker counterpart amounts to be 10.

(ii) For the benchmark scenario, it is assumed that  $\rho_{\Delta} \equiv 0$  in (12) and investment-specific technology shocks are a sole driving force. The representative agent’s anticipation horizon in the benchmark case equals 4 (quarters).

(iii) The present version of Danthine and Donaldson (2015) is identical to their optimal case except the assumption of fixed labor supply. The standard deviation for the model’s Prescottian TFP Shocks takes the value of 0.72% (Cooley and Prescott, 1995).

(iv) Positive numbers indicate welfare gains, while negative ones mean welfare losses.

(v) Excerpted from Kim and Joo (2019).

(vi) Excerpted from Kim and Kim (2014).

Table 2 shows that costs of business cycles in the benchmark delegated management model, i.e., in Proposition 1, are substantial: the welfare effects from eliminating Pigouvian business cycles are about one order of magnitude larger than the delegated management economy characterized by standard Prescottian cycles (the DD entry), virtually isomorphic to Lucas’ original estimate,<sup>9</sup> and even the business cycle model with the aforementioned twin stochastic trends

<sup>9</sup>As indicated in Table 2, the latter scenario makes the twin assumptions of the representative consumer’s fixed labor supply and Prescottian contemporaneous TFP shocks.

(the RBC-Fisher entry). Note that the benchmark model allows for investment-specific Pigouvian shocks only. The suboptimal economy driven by the manager's (implied) suboptimal contracts — the column headed by the phrase “Concentrated Equity Ownership” — appears to produce substantial welfare costs, but about a half of the benchmark counterpart through the Prescottian channel of supply shocks. The economic narrative behind these large welfare effects is that aggregate fluctuations are created by autonomous changes in the private agent's expectations, including the manager's counterparts, in response to news about fundamentals and not due to *real* variations in fundamentals.<sup>10</sup>

**Remark 3.** *An important caveat of the present model is that the cost of Pigouvian cycles, in principle, is not necessarily robust to the twin factors of the variability of endogenous labor supply and a balanced-growth-path condition, as thoroughly discussed in Donaldson et al. (2022). The former factor, simply referred to as the “mean effect” of business cycles, plays a pivotal role in determining an economy's overall cost of fluctuations: this mean effect, as counterintuitive as it appears, often creates the welfare benefit of the business cycle, rather than its welfare cost (Cho et al., 2015).*

#### 4. CONCLUDING REMARKS

The present study offers a simple model of delegated management within the general equilibrium context of Pigouvian cycles. In sharp contrast to leading microeconomic models of moral hazard, the general equilibrium macroeconomic model noted here has several distinctive features. While even first-best executive compensation contracts may produce economy-wide welfare losses more than one order of magnitude larger than the Lucas (1987, 2003) cost-of-business-cycle estimate, the contract's pay-for-luck phenomenon proves to be irrelevant to any distortions of social welfare. Relatedly, the soaring CEO-to-worker pay ratio, emblematic of the present-day US economy's polarization, is also found to be irrelevant to social welfare. An introduction of the manager's unobservable efforts under the heading “private information” does not change the latter conclusion as well, since Prescott and Townsend (1984) securitization scheme vis-à-vis private

<sup>10</sup>Following Donaldson et al. (2022), the further line of economic reasoning is that the representative consumer ends up being in a position to choose between the Pigouvian world, where his consumption stream is not necessarily related to fundamentals and vulnerable to news, whether good or bad, and the Lucasian world, where the planner's full commitment technology produces a fundamentals-based consumption stream. The latter world proves to be the stand-in consumer's preferred choice, leading to welfare costs about one-order magnitude greater than Lucas (1987, 2003) counterpart.

information within the present context will wash out any private-information idiosyncratic shocks at the aggregate level.

## REFERENCES

- Andreasen, M. (2012). “On the Effects of Rare Disasters and Uncertainty Shocks for Risk Premia in Non-linear DSGE Models,” *Review of Economic Dynamics* 15, 295-316
- Bertrand, M., and S. Mullainathan (2003). “Enjoying the Quiet Life? Corporate Governance and Managerial Preferences,” *Journal of Political Economy* 111, 1043–1075.
- Chari, V. V., Kehoe, P. J., and E. R. McGrattan (2007). “Business Cycle Accounting,” *Econometrica* 75, 781-836.
- Cho, J.O., Cooley, T.F., and H. S. E. Kim (2015). “Business Cycle Uncertainty and Economic Welfare,” *Review of Economic Dynamics* 18, 185-200.
- Christiano, L. J., Eichenbaum, M., and C. L. Evans (2005). “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,” *Journal of Political Economy* 113, 1-45.
- Christiano, L. J., Ilut, C., Motto, R., and M. Rostagno (2010). “Monetary Policy and Stock Market Booms,” NBER Working Paper 16402.
- Cooley, T. F. and E. C. Prescott (1995). “Economic Growth and Business Cycles,” in *Frontiers of Business Cycle Research*, eds. T. F. Cooley, Princeton University Press, 1-38.
- Danthine, J. P. and J. B. Donaldson (2015), “Executive Compensation: A General Equilibrium Perspective,” *Review of Economic Dynamics* 18, 269-286.
- Donaldson, J. B. and H. S. E. Kim (2023). “Executive Compensation: Inconvenient Truths,” Working Paper.
- Donaldson, J. B., Joo, S., Kim, H. S. E., and Y. Lee (2022). “The Costs of Pigouvian Cycles,” Working Paper.
- Fisher, J. (2006). “The Dynamic Effects of Neutral and Investment-specific Technology Shocks,” *Journal of Political Economy* 114, 413-451.
- Garvey, G. and T. Milbourn (2006). “Asymmetric Benchmarking in Compensation: Executives Are Rewarded for Good Luck but not Penalized for Bad,” *Journal of Financial Economics* 82, 197-225.

- Greenwood, J., and Hercowitz, Z., and P. Krusell (2000). "The Role of Investment-specific Technological Change in the Business Cycle," *European Economic Review* 44, 91-115.
- Guvenen, F. (2009). "A Parsimonious Model for Asset Pricing," *Econometrica* 77, 1711-1750.
- Hansen, G. (1985). "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics* 16, 309-327.
- Jaimovich, N. and S. Rebelo (2009). "Can news about the future drive the business cycle?," *American Economic Review*, 99, 1097-1118
- Kaltenbrunner, G. and L. Lochstoer (2010). "Long Run Risk through Consumption Smoothing," *Review of Financial Studies* 23, 3190-3224.
- Kim, H. S. E. (2023). "A Model of Delegated Management," *Korean Social Science Journal* 50, 19-37.
- Kim, J. H. and H. S. E. Kim (2014). "The Costs of Business Cycles with Market Incompleteness I," *Journal of Economic Theory and Econometrics*, 25, 40-59.
- Kim, H.S. and S. Joo (2019). "Revisiting the Cost of Business Cycles: A Demand Shock Channel," *Journal of Economic Theory and Econometrics* 3, 55-75.
- Kydland, F. E. and E. C. Prescott (1982). "Time to Build and Aggregate Fluctuations," *Econometrica* 50, 1345-1370.
- Lucas, R. E. (1987) "Models of Business Cycles Basil Blackwell," *New York*.
- Lucas, R. E. (2003) "Macroeconomic Priorities," *American Economic Review* 93, 233-247.
- Mehra, R. and E. C. Prescott (1985). "The Equity Premium: A Puzzle," *Journal of Monetary Economics* 22, 145-161.
- Prescott, E. C. (1986). "Theory Ahead of Business-Cycle Measurement," *Carnegie-Rochester Conference on Public Policy* 24, 11-44.
- Prescott, E. C. and R. M. Townsend (1984) "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard," *Econometrica* 52, 21-45.



Schmitt-Grohé, S. and M. Uribe (2012). “What’s News In Business Cycles,”  
*Econometrica* 80, 2733-2764.

## TECHNICAL APPENDIX

## A. PARETO-OPTIMAL ALLOCATION

In this section, we characterize the first best allocation for the economy discussed so far. The recursive representation of the problem (9) is:

$$V^{CP}(\Omega_t) = \max_{\{c_t^s, c_t^m, n_t^s, i_t, v_t\}} \left[ \begin{array}{l} \theta \mu u(c_t^m - bc_{t-1}^m) + u(c_t^s - bc_{t-1}^s) - H(n_t^s) \\ + \lambda_t [f(z_t, v_t k_t, n_t^s) - \mu c_t^m - c_t^s - i_t] \\ + \beta \mathbb{E}(V^{CP}(\Omega_{t+1}) | \Omega_t) \end{array} \right] \quad (15)$$

s.t.

$$k_{t+1} = [1 - \delta(v_t)]k_t + i_t [1 - \phi(\frac{i_t}{i_{t-1}})] \quad (16)$$

where  $\lambda_t$  is the multiplier associated with (16).

The necessary and sufficient first order conditions to the above problem(15) is:

$$\begin{aligned} c_t^m &: \theta \mu u_c(c_t^m - bc_{t-1}^m) - \mu \lambda_t + \beta \mathbb{E}(V_{c_t^m}^{CP} | \Omega_t) = 0 & (17) \\ c_t^s &: u_c(c_t^s - bc_{t-1}^s) - \lambda_t + \beta \mathbb{E}(V_{c_t^s}^{CP} | \Omega_t) = 0 \\ n_t^s &: \lambda_t f_{n_t^s} - H_{n_t^s}(n_t^s) = 0 \\ v_t &: \lambda_t f_{v_t} k_t + \beta \mathbb{E}(V_{k_{t+1}}^{CP} \frac{\partial k_{t+1}}{\partial v_t} | \Omega_t) = 0 \\ i_t &: (-1)\lambda_t + \beta \mathbb{E}(V_{i_t}^{CP} + V_{k_{t+1}}^{CP} \frac{\partial k_{t+1}}{\partial i_t} | \Omega_t) = 0 \\ c_{t-1}^m &: V_{c_{t-1}^m}^{CP} = \theta \mu u_c(c_t^m - bc_{t-1}^m)(-b) \\ c_{t-1}^s &: V_{c_{t-1}^s}^{CP} = u_c(c_t^s - bc_{t-1}^s)(-b) \\ i_{t-1} &: V_{i_{t-1}}^{CP} = \beta \mathbb{E}(V_{k_{t+1}}^{CP} \frac{\partial k_{t+1}}{\partial i_{t-1}} | \Omega_t) \\ k_t &: V_{k_t}^{CP} = \lambda_t f_{k_t} v_t + \beta \mathbb{E}(V_{k_{t+1}}^{CP} \frac{\partial k_{t+1}}{\partial k_t} | \Omega_t) \end{aligned}$$

where  $\frac{\partial k_{t+1}}{\partial i_t} = 1 - \phi(\frac{i_t}{i_{t-1}}) - \phi'(\frac{i_t}{i_{t-1}}) \frac{i_t}{i_{t-1}}$ ,  $\frac{\partial N_{t+1}}{\partial v_t} = -\delta'(v_t)k_t$ ,  $\frac{\partial k_{t+1}}{\partial k_t} = 1 - \delta(v_t)$ , and  $\frac{\partial k_{t+1}}{\partial i_{t-1}} = \phi'(\frac{i_t}{i_{t-1}})(\frac{i_t}{i_{t-1}})^2$  respectively.

We summarize the Euler conditions for the central planner's problem:

$$\begin{aligned}
(\text{P1}): \quad & \theta \lambda_t^m = \lambda_t & (18) \\
(\text{P2}): \quad & \lambda_t f_{n_t^s} - H_{n_t^s}(n_t^s) = 0 \\
(\text{P3}): \quad & \lambda_t \frac{f_{v_t}}{\delta'(v_t)} = \beta \mathbb{E}(\lambda_{t+1} [f_{k_{t+1}} v_{t+1} + (1 - \delta(v_{t+1})) \frac{f_{v_{t+1}}}{\delta'(v_{t+1})}] \mid \Omega_t) \\
(\text{P4}): \quad & 1 = \beta \mathbb{E}(\frac{\lambda_{t+1}}{\lambda_t} \frac{f_{v_{t+1}}}{\delta'(v_{t+1})} \phi'(\frac{i_{t+1}}{i_t}) (\frac{i_{t+1}}{i_t})^2 \mid \Omega_t) + \frac{f_{v_t}}{\delta'(v_t)} [1 - \phi(\frac{i_t}{i_{t-1}}) - \phi'(\frac{i_t}{i_{t-1}}) \frac{i_t}{i_{t-1}}] \\
(\text{P5}): \quad & \lambda_t^m = u_c(c_t^m - bc_{t-1}^m) - b\beta \mathbb{E}(u_c(c_{t+1}^m - bc_t^m) \mid \Omega_t) \\
(\text{P6}): \quad & \lambda_t = u_c(c_t^s - bc_{t-1}^s) - b\beta \mathbb{E}(u_c(c_{t+1}^s - bc_t^s) \mid \Omega_t)
\end{aligned}$$

### B. PROOF OF THEOREM 1

Under the contract of form 10, the recursive formulation of delegated manager's problem is:

$$V^m(\Omega_t^m) = \max_{\{i_t, n_t^s, v_t\}} [u(c_t^m - bc_{t-1}^m) + \lambda_t^m [\mathbb{C}^{PO}(i_t, n_t, v_t; \Omega_t^m) - c_t^m] + \beta \mathbb{E}(V^m(\Omega_{t+1}^m) \mid \Omega_t^m)]$$

$$\begin{aligned}
\text{s.t. } k_{t+1} &= [1 - \delta(v_t)]k_t + i_t [1 - \phi(\frac{i_t}{i_{t-1}})] \\
n_t^m &= 1
\end{aligned}$$

The necessary and sufficient first order conditions to the above problem can be written as

$$\begin{aligned}
c_t^m &: u_c(c_t^m - bc_{t-1}^m) - \lambda_t^m + \beta \mathbb{E}(V_{c_t}^m \mid \Omega_t^m) = 0 & (19) \\
n_t^s &: \phi \lambda_t^m (f_{n_t^s} - w_t) = 0 \\
v_t &: \phi \lambda_t^m f_{v_t} k_t + \beta \mathbb{E}(V_{k_{t+1}}^m \frac{\partial k_{t+1}}{\partial v_t} \mid \Omega_t) = 0 \\
i_t &: \phi(-1) \lambda_t^m + \beta \mathbb{E}(V_{i_t}^m + V_{k_{t+1}}^m \frac{\partial k_{t+1}}{\partial i_t} \mid \Omega_t) = 0 \\
c_{t-1}^m &: V_{c_{t-1}}^m = u_c(c_t^m - bc_{t-1}^m)(-b) \\
i_{t-1} &: V_{i_{t-1}}^m = \beta \mathbb{E}(V_{k_{t+1}}^m \frac{\partial k_{t+1}}{\partial i_{t-1}} \mid \Omega_t) \\
k_t &: V_{k_t}^m = \phi \lambda_t^m f_{k_t} v_t + \beta \mathbb{E}(V_{k_{t+1}}^m \mid \Omega_t) \frac{\partial k_{t+1}}{\partial k_t}
\end{aligned}$$

where  $\frac{\partial k_{t+1}}{\partial i_t} = 1 - \phi\left(\frac{i_t}{i_{t-1}}\right) - \phi'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}$ ,  $\frac{\partial N_{t+1}}{\partial v_t} = -\delta'(v_t)k_t$ ,  $\frac{\partial k_{t+1}}{\partial k_t} = 1 - \delta(v_t)$ , and  $\frac{\partial k_{t+1}}{\partial i_{t-1}} = \phi'\left(\frac{i_t}{i_{t-1}}\right)\left(\frac{i_t}{i_{t-1}}\right)^2$  respectively. The Euler equations for the delegated manager's problem can be written as:

$$\begin{aligned}
\lambda_t^m &= u_c(c_t^m - bc_{t-1}^m) - b\beta\mathbb{E}(u_c(c_{t+1}^m - bc_t^m) \mid \Omega_t) & (20) \\
w_t &= f_{n_t^s} \\
\lambda_t^m \frac{f_{v_t}}{\delta'(v_t)} &= \beta\mathbb{E}(\lambda_{t+1}^m [f_{k_{t+1}} v_{t+1} + (1 - \delta(v_{t+1})) \frac{f_{v_{t+1}}}{\delta'(v_{t+1})}] \mid \Omega_t) \\
1 &= \beta\mathbb{E}\left(\frac{\lambda_{t+1}^m}{\lambda_t^m} \frac{f_{v_{t+1}}}{\delta'(v_{t+1})} \phi'\left(\frac{i_{t+1}}{i_t}\right) \left(\frac{i_{t+1}}{i_t}\right)^2 \mid \Omega_t\right) \\
&+ \frac{f_{v_t}}{\delta'(v_t)} \left[1 - \phi\left(\frac{i_t}{i_{t-1}}\right) - \phi'\left(\frac{i_t}{i_{t-1}}\right) \frac{i_t}{i_{t-1}}\right]
\end{aligned}$$