Hidden Saving and In-kind Transfers
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Abstract  This paper revisits the claim that public provision of in-kind transfer is more efficient than transfers in cash. A simple job search model suggests that moral hazard would become more severe if recipients can save the transfer payment privately (the hidden saving problem), inducing them to make less effort to find jobs (that is, double deviation problem). We show that because the hidden saving problem always exists, economic efficiency requires overprovision of in-kind transfers and undersupply of cash grants. Our finding suggests that saving is not a virtue for government transfers.

Keywords  In-kind transfers, cash transfers, moral hazard, hidden saving

JEL Classification  H42, H31

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1. INTRODUCTION

With a few exceptions, in-kind transfers are rarely justified on the basis of efficiency. Cash transfers are considered more efficient than in-kind transfers, because the latter constrain the behavior of the recipients.\(^1\) Despite this simple logic, in-kind transfer programs are widely used for redistribution purposes. On average, countries from the Organisation for Economic Co-operation and Development (OECD) provide about 50 percent of their social aid through in-kind transfers, including housing, education, child care, health care, and food subsidies (Marical et al. 2006). Traditional explanations for the existence of in-kind transfers include paternalism (donors care about recipients’ consumption of specific goods), self-targeting (non-targeted individuals are less willing to take in-kind benefits than cash grants), and rent-seeking (industries support in-kind programs related to their products).\(^2\)

On the contrary, some literature posits that in-kind transfers increase economic efficiency relative to cash transfers.\(^3\) One argument centers around the inability of a government to commit to no bailouts—that is, anticipation of future transfers undermines the recipient’s effort to bail himself out (Buchanan 1977; Lindbeck and Weibull 1988; Bruce and Waldman 1991; Coate 1995; Pedersen 2001; Lagerl öf 2004; Hagen 2006). The Samaritan’s dilemma arises because recipients of transfers are entitled to receive future transfers only if they remain poor. Provision of in-kind benefits, such as job training, is said to reduce the time-inconsistency problem. As we will show, however, in-kind transfers can enhance efficiency even if the government is able to commit not to bail out today’s recipients in the future.

Another strand of argument suggests that in-kind transfers improve tax efficiency via increased labor supply (Gahvari 1994, 1995; Currie and Gahvari 2008; Blomquist et al. 2010). Over providing in-kind transfers, such as child care and health care, can potentially stimulate labor supply, offsetting the extant deadweight loss from income tax.\(^4\) Moreover, increases in the labor supply will generate extra tax revenues, which can be used to finance more expenditures,

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\(^1\)Recipients of in-kind benefit programs are not typically allowed to resell their allotments. This makes it difficult for the recipients to equalize the marginal rate of substitution and the price ratio, which results in a “corner” solution.

\(^2\)See, for example, Blackorby and Donaldson (1988); Benson and Mitchell (1988); Mulligan and Philipson (2000); Currie and Gahvari (2008); and Rosen and Gayer (2010, p. 272). Self-targeting can be regarded as efficiency-enhancing to the extent that mimicking behaviors of non-targeted individuals make cash grants an inefficient tool.

\(^3\)For a more general survey, see Currie and Gahvari (2008).

\(^4\)Giving cash transfers may not increase labor supply due to the income effect.
positively affecting aggregate welfare. Currie and Gahvari (2008), however, noted that time horizon matters, because in-kind transfers affect labor supply in the long run.

In this paper, we provide another robust explanation for why in-kind transfers can be more efficient than cash transfers. We consider a two-period job search model with two commodities—cash and goods in-kind—and two types of agents—unemployed and employed. In the first period, the unemployed agent searches for a job. The probability of the agent finding a job in the second period depends on the search effort. In the model, a benevolent social planner chooses the consumption of both types of agents. The planner observes whether an agent is employed, but the effort level that determines the probability of employment is private information not available to the planner. Thus, the consumption assigned to the unemployed agent must depend on employment status (in the second period), rather than search effort (in the first period). This asymmetric information makes it difficult for the planner to control the agent’s moral hazard problem.

Our central insight is that moral hazard would become more severe if the unemployed agent saves the transfer payment without the planner’s knowledge. This hidden saving problem induces agents to make less efforts to find jobs. We show that the hidden saving problem always exits in the sense that when the agent deviates from making high effort, he always saves the transfer payment—a phenomenon known as the double deviation problem (Chien and Song 2013). The agent cannot, however, save in-kind transfers because the planner can easily monitor the agent’s consumption.

The incentive-constrained efficient allocation (i.e., the solution to the planner’s optimization problem) indicates that over providing in-kind transfers is necessary for efficiency. Intuitively, the hidden saving problem distorts the search effort that the planner wants to implement. The planner then wants to provide

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5 More precisely, cash denotes the goods that, once publicly provided, can be resold and would be equivalent to a cash transfer.

6 The employed agent, on the other hand, becomes unemployed with an exogenously given probability.

7 Both agents consume cash and in-kind goods during each period.

8 One limitation of our model is that it assumes that all in-kind transfers are safe from the hidden savings problem. However, some forms of in-kind transfers can be subject to the hidden saving. We focus more on those in-kind benefits whereby such possibilities are limited, such as job training and child care.

9 Of course, the planner cannot provide only in-kind transfers, as this would decrease efficiency substantially by an extreme departure of the marginal rate of substitution from the first best marginal rate of substitution.
more of the goods in kind, which the agent cannot save. Our finding can explain why some governments are more likely to give cash transfers to the elderly age group, and in-kind transfers to families with children.

Our result is related to, but different from, the previous literature. In particular, we explicitly incorporate search effort into the Samaritan’s dilemma model. The Samaritan’s dilemma suggests that recipients would use the cash transfer in an inefficient manner, such as saving too little. For instance, an intertemporal equilibrium is inefficient if strategic undersaving induces the donor to give larger support than otherwise (Lindbeck and Weibull 1988). Lagerlöf (2004), however, noted that if the donor is uncertain about the recipient’s need, the recipient will strategically save more to signal that he is in great need. In the present analysis, (hidden) saving of cash transfers leads to an inefficiently low level of search effort, which is not directly observable. Our paper thus shows that public provision of in-kind transfer is more efficient than cash transfers even if there is no time-inconsistency problem. In addition, we consider the moral hazard problem in a dynamic setting for the tax efficiency literature. The tax efficiency argument claims that in-kind transfers substitute leisure, thus stimulating work effort. In the present study, in-kind transfers increase future labor supply by stimulating job search effort.

Our results are also related to the optimal tax literature (Diamond and Mirrlees 1971a, b; Stiglitz and Dasgupta 1971; Atkinson and Stiglitz 1972). These studies argue that even if labor supply cannot be taxed properly (because earning abilities are difficult to observe), a government can implicitly tax the labor supply by taxing commodities that complement the labor supply. Our results suggest that if high effort in job search is not directly enforceable, the government can instead subsidize the commodity that is less susceptible to the hidden savings problem.

This paper is organized as follows. In Section 2, we offer a theoretical model of efficient allocation, and provide an explanation for why in-kind transfers could enhance efficiency. In Section 3, we discuss our results and conclude with implications for social transfers in practice. Proofs are provided in the Appendix.

2. MODEL

To examine the real collective decision making, public choice theory models a government policy of redistribution alternatively as a result of competition between interest groups, the optimization behavior of a social planner, or the

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10 Even food stamps cannot be saved as they will expire and are prohibited from resale.
choice of a Leviathan (Lee et al. 2013). Our study employs a simple social planner model to characterize the incentive-constrained efficient allocation because this approach easily incorporates the pre-eminent effect of a hidden saving in examining the relationship between key variables. Methodologically, our model is related to Chien and Song’s (2013) principal-agent model, in which an agent makes an effort and the principal awards the agent with outcome-contingent payments.\footnote{They showed that if the principal can award two types of commodities, the principal prefers the commodity that does not suffer from a hidden saving problem. Our model differs from that of Chien and Song in that (i) we consider a welfare-maximizing policy (instead of the profit maximization of a principal), and (ii) we have two types of populations, the unemployed and the employed, in order to derive a discrepancy in the marginal rate of substitutions between the two groups (instead of a single agent and the given price ratio).}

Consider a two-period economy with a continuum of agents with a unit mass in each period. There are two types of agents in the initial period: an unemployed type—who may find a job in the next period—and an employed type—who may lose a job in the next period. The population share of the unemployed is $\mu$ ($0 < \mu < 1$) in the first period. There are two commodities, $x$ and $y$, in the first period, and $X$ and $Y$ in the second period. Note that $x$ and $X$ ($y$ and $Y$) simply distinguish the consumption in the two periods.

The unemployed type makes an effort, $e$, to search for a job during the first period. This search effort is the agent’s private information and determines the distribution of the outcome in the second period. For simplicity, we assume that the unemployed agent can exert either a high or low effort for their job search. More specifically, there are two effort levels, $e_H$ and $e_L$, with $e_L < e_H$. The outcome $s$ is either “found a job” or “not found a job,” denoted by “f” and “n.” We denote the probability of finding a job by $P(f|e)$ when the agent makes effort $e \in \{e_H, e_L\}$. In the second period, the outcome $s \in \{f, n\}$ is realized.

An employed type, who makes no search effort in the first period, either be-
comes unemployed with probability $Q(n)$ or remains employed with probability $Q(f) = 1 - Q(n)$.

The $x_i$ and $y_i$ denote the first-period consumption of an agent $i \in \{o, j\}$ (where $o$ and $j$ denote the unemployed and the employed in the first period, respectively). $X_i(s)$ and $Y_i(s)$ denote the second-period consumption at state $s \in \{f, n\}$.

An agent has a hidden saving technology for goods $x_i$, but not for goods $y_i$. That is, the agent can transfer the first-period consumption of $x_i$ into the second period without the planner’s knowledge. Let $\sigma_k$ be the unemployed type’s optimal saving when the agent makes effort $e_k$, $k \in \{H, L\}$. The interest rate is assumed to be zero.\(^{13}\) Given the planner’s resource allocation $(x_o, y_o, X_o(f), Y_o(f), X_o(n), Y_o(n))$, the unemployed agent’s maximized utility is

$$\max_{e, \sigma} \left[ u(x_o - \sigma) + v(y_o) - c(e) + \sum_s [U(X_o(s) + \sigma) + V(Y_o(s))] P(s|e) \right]$$  \label{eq:1}

where $u(\cdot) + v(\cdot)$ and $U(\cdot) + V(\cdot)$ are temporal utility functions for periods 1 and 2, respectively, and $c(e)$ is the agent’s cost of search effort. The utility functions $u(\cdot)$ and $U(\cdot)$ (and $v(\cdot)$ and $V(\cdot)$) simply distinguish periods 1 and 2. To simplify the analysis, we assume that the utility functions are strictly concave and additively separable; both types of agents have identical utility functions; and there is no discount between periods. These simplifications do not alter our results qualitatively.

The planner implements a high search effort $e_H$ and seeks to prevent the unemployed type’s deviation to a low search effort.\(^{14}\) Thus, the “incentive compatibility constraint” for the unemployed agent is given by

$$\langle e_H, \sigma_H \rangle \in Eq.(1)$$

For simplicity, but without a loss of generality, we normalize $\sigma_H^* = 0$;\(^{15}\) a hidden saving problem exists because $\sigma_L^* > 0$ (as will be formally derived in

\(^{13}\)A non-zero interest rate would not change our results qualitatively.

\(^{14}\)To make our analysis meaningful, we assume that the planner’s benefit of implementing a high effort exceeds the combined cost of allocating the resources and of implementing the incentive compatibility.

\(^{15}\)Suppose, under optimal allocation $(x_o^*, y_o^*, X_o^*(\cdot), Y_o^*(\cdot))$, that the unemployed agent chooses nonzero $\sigma_H$. However, from the new allocation $(x_o^* - \sigma_H^*, y_o^*, X_o^*(\cdot) + \sigma_H^*, Y_o^*(\cdot))$, it is readily seen that this implements zero saving, as the allocation does the saving for the unemployed agent. It is trivial that this new allocation implements high effort $e_H$. Thus, we can normalize $\sigma_H^* = 0$ without a loss of generality. Alternatively, even if $\sigma_H^* \neq 0$, the results will not change qualitatively—that is, $\sigma_L^* > \sigma_H^*$ will be derived.
Proposition 2). Alternatively, we can directly show $\sigma_L^* > \sigma_H^*$ at the cost of more complicated algebra. Intuitively, given that $P(f|e_L) < P(f|e_H)$, a consumption-smoothing agent with $e_L$ will save more than would be the case with $e_H$.\(^{16}\) Conversely, an agent who cannot save for the future will make $e_H$ to increase $P(f|e)$. The hidden saving and the incentive compatibility constraints can be summarized as:

\[
\begin{align*}
    u'(x_o) &= \sum_s U'(X_o(s))P(s|e_H) \tag{2} \\
    u'(x_o - \sigma_L) &= \sum_s U'(X_o(s) + \sigma_L)P(s|e_L) \\
    u(x_o) + v(y_o) - c(e_H) + \sum_s [U(X_o(s)) + V(Y_o(s))]P(s|e_H) \geq u(x_o - \sigma_L) + v(y_o) - c(e_L) + \sum_s [U(X_o(s) + \sigma_L) + V(Y_o(s))]P(s|e_L) \tag{4}
\end{align*}
\]

Equation (2) states that, for the unemployed to choose $\sigma_H = 0$, the consumption $(x,X(\cdot))$ must satisfy the Euler equation, which equates the first-period marginal utility and the second-period marginal utility. Equation (3) describes the optimal saving $\sigma_L$, given low effort $e_L$.\(^{17}\) With $\sigma_L$ derived from (3), the inequality shown in (4) indicates the incentive compatibility constraint for the effort level.

Additionally, we have the following resource constraints:

\[
\begin{align*}
    \mu x_o + (1 - \mu) x_j + \mu \sum_s X_o(s)P(s|e_H) + (1 - \mu) \sum_s X_j(s)Q(s) & \leq w^x + w^X \tag{5} \\
    \mu y_o + (1 - \mu) y_j + \mu \sum_s Y_o(s)P(s|e_H) + (1 - \mu) \sum_s Y_j(s)Q(s) & \leq w^y + w^Y \tag{6}
\end{align*}
\]

where $w^x$ and $w^y$ ($w^X$ and $w^Y$) are the given resources of goods $x$ and $y$ ($X$ and $Y$) in period 1 (period 2). Note that the left sides of the constraints indicate the allocations made by the planner, and the right sides are the available resources.\(^{18}\)

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\(^{16}\) Saving reduces the risk from staying unemployed in the future.

\(^{17}\) If the agent deviates to a low effort $e_L$, the probability distribution $P(s|e)$ will change, adjusting the optimal saving.

\(^{18}\) There is one resource constraint for each good. However, even with temporal resource constraints (i.e., four resource constraints for the two goods and the two periods), the qualitative results will remain the same (results available from the authors upon request).
Thus the planner’s maximization problem is given by:

\[
\max_{x_i, y_i} \beta \left[ u(x_o) + v(y_o) - c(e_H) + \sum_i [U(X_i(s)) + V(Y_i(s))] P(s|e_H) \right] \\
+ (1 - \beta) \left[ u(x_j) + v(y_j) + \sum_i [U(X_j(s)) + V(Y_j(s))] Q(s) \right]
\]

subject to (2), (3), (4), (5), and (6)

where the multipliers of the constraints (2), (3), (4), (5), and (6) are denoted by \( \mu, \lambda_L, \mu, \alpha, p_x, \) and \( p_y, \) respectively.\(^{19}\)

**Definition 1.** The incentive-constrained efficient allocation is the solution of the planner’s problem (7) subject to (2) through (6).

Note that \( \beta \) denotes the distributional weight of the unemployed type, which is not necessarily equivalent to the population share \( \mu.\) The planner’s problem does not include the Euler equations for the employed type, but we can show that the Euler equations for both \( x_j \) and \( y_j \) are, in fact, derived as the conditions for the incentive-constrained efficient allocation. From the first order conditions (shown in (A1) through (A8) in the Appendix), we obtain:\(^{20}\)

\[
\begin{align*}
\frac{d}{dx} u(x_o) &= U'(X_o(s)) \\
\frac{d}{dy} v(y_o) &= V'(Y_o(s)) \\
\frac{d}{dx} u(x_o) &\neq U'(X_o(s)) \\
\frac{d}{dy} v(y_o) &\neq V'(Y_o(s))
\end{align*}
\]

In the first two equalities in (8), efficient allocation ensures that the employed agent achieves consumption smoothing across periods and states—now satisfying the Euler equations for \( (x_j, X_j(s)) \) and \( (y_j, Y_j(s)) \). The unemployed agent, however, is not able to smooth consumption across states. The last two terms in (8) imply that marginal utilities of the second period consumption must differ by states, although the unemployed agent still smooths expected consumption across periods.\(^{21}\) Intuitively, the employed agent—without moral hazard problem—does not face the incentive compatibility constraint. On the contrary,

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\(^{19}\)The first three multipliers contain probabilities \( \mu, \) so that the shadow values are in the same metric.

\(^{20}\)See the Appendix for the proof.

\(^{21}\)From (2), \( u'(x_o) = P(f|e_H)U'(X_o(f)) + P(n|e_H)U'(X_o(n)) \). Thus, if \( u'(x_o) \neq U'(X_o(s)), U'(X_o(f)) \) and \( U'(X_o(n)) \) cannot be the same.
informational asymmetry for the unemployed type prevents the equal treatment of states.

Simplifying the first-order conditions further, we derive the following

\[ \frac{p_x}{u'(x_o)} = \frac{\beta}{\mu} + \alpha \left( 1 - \frac{u'(x_o - \sigma_L)}{u'(x_o)} \right) \]  

(9)

\[ \frac{p_x}{U'(X_o(s))} = \frac{\beta}{\mu} + \alpha \left( 1 - \frac{U'(X_o(s) + \sigma_L)}{U'(X_o(s))} \frac{P(s|e_L)}{P(s|e_H)} \right) \]  

(10)

where \( \alpha > 0 \) and \( p_x > 0 \).\(^{22}\)

See the Appendix for proofs of all propositions.

**Remark:** For simplicity, we assume \( \mu = \beta \). If there were no hidden saving problem (i.e., the agent cannot save cash privately), (9) and (A5)—see p. 17—become: \( u'(x_o) = p_x = u'(x_j) \), which implies that the marginal utility of \( x_o \) and \( x_j \) are equivalent to the shadow value of the resource constraint for \( x \), or \( p_x \). Note that \( p_x \) reflects the true value of an additional resource \( x \). In the presence of the hidden saving problem (i.e., \( \sigma_L > 0 \)), however, (9) and (A5) indicate that:

\[ u'(x_o) > p_x = u'(x_j). \]  

(11)

The intuition for this is straightforward: If additional \( x \) is given to the unemployed agent, \( o \), the hidden saving problem will break the incentive compatibility constraint. Thus, the value of the additional resource \( x \) must be smaller than \( u'(x_o) \).

Equation (10) tells us that if the hidden saving problem does not exist, the planner simply assigns the second period consumption based on outcome. That is, setting \( \sigma_L = 0 \), (10) indicates that \( X_o(f) \) is greater than \( X_o(n) \).\(^{23}\) This practically rules out the Samaritan’s dilemma as the planner fully commits to the allocation in period 2.

We show, however, that the hidden saving problem—instead of hidden borrowing—always exists.

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\(^{22}\)Lemma 1 in the Appendix shows that \( \lambda_H = \lambda_L = 0 \). Even in this case, conditions (2) and (3) are binding. Formally, we can formulate an alternative planner’s problem without conditions (2) and (3), and show that the two conditions are not satisfied. See the Appendix for detailed discussions.

\(^{23}\)Note that \( P(f|e_L)/P(f|e_H) < 1 \) and \( P(n|e_L)/P(n|e_H) > 1 \).
Proposition 2. $\sigma_L > 0$.

Proposition 2 is our main finding, and we have claimed that the presence of hidden savings increases the moral hazard problem. Intuitively, given a concave utility function, saving reduces the utility difference between “f” and “n” in period 2. Thus, implementing a high search effort, $e_H$, is more costly when the unemployed can save privately.\(^{24}\)

In the presence of the hidden saving problem and moral hazards, the incentive-constrained efficiency requires that two types of agents have different marginal rates of substitution between $x$ and $y$.

Proposition 3. The incentive-constrained efficient allocation implies

$$
\frac{u'(x_o)}{v'(y_o)} > \frac{u'(x_o)}{v'(y_o)} \left[ 1 + \frac{\mu}{\bar{\beta}} \alpha \left( 1 - \frac{u'(x_o - \sigma_L)}{u'(x_o)} \right) \right] = \frac{p_x}{p_y} = \frac{u'(x_j)}{v'(y_j)}
$$

Proposition 3 indicates that the planner provides relatively less $x$ (cash transfers) and more $y$ (in-kind transfers) to the unemployed than would be the case with the employed. The inequality in (12) would turn into equality either if the hidden saving problem is not present (i.e., $\sigma_L = 0$ in (12)), or if moral hazard is not a concern (i.e., $\alpha = 0$ in (12)). This would then equalize marginal rates of substitution between $x$ and $y$ across both types of agents. Note also that even if the distributional weight assigned to the unemployed agent, $\beta$, is 1 (i.e., the planner cares only about the unemployed), the inequality holds as long as both $\sigma_L$ and $\alpha$ are positive.

3. DISCUSSIONS AND CONCLUSION

Figure 1 illustrates a transfer program with the hidden saving problem. Reflecting the theoretical model, the preferences depend on the composite consumption good, $x$, and a good subject to in-kind transfers, $y$. Unemployed agents prefer a positive saving, $\sigma$, other things equal. The original budget constraint is represented by $EF$ on the $xy$-plane. Cash transfers shift the budget line upward to $GH$. Note that $FH$ measures the amount of cash transfer, assuming that the price of $x$ is unity. In-kind transfers of equal cost shift the budget line to

\(^{24}\)Formally, we can show that $|U(X_o(f)) - U(X_o(n))| > |U(X_o(f) + \sigma) - U(X_o(n) + \sigma)|$ for $\sigma > 0$, since utility function $U(\cdot)$ is concave. Thus, inducing $e_H$ (i.e., $\sigma_H = 0$) would require a greater distance between $U(X_o(f))$ and $U(X_o(n))$. But, this implies a greater risk-taking for the unemployed, and the overall level of $U(X_o(s))$ must increase so as to compensate for the additional risk (i.e., to satisfy the participation constraint).
The agent clearly prefers cash transfers—choosing point $A$—to in-kind transfers—with the choice being at point $B$. Under in-kind transfers, $y$ is over provided.

However, if the agent saves some or all of the cash transfers, the current period’s budget constraint under the cash transfers shifts from $GH$ forward up to $G'H'$. The diagram shows that the unemployed agent can pick point $C$ over point $A$, implying a low effort (since a positive saving induces a low effort level). Thus, in order to enforce high effort, the planner should employ an in-kind transfer program, limiting the agent’s consumption choice at point $B$.

Our theoretical results are in line with the facts that the U.S. government is...
more likely to give cash transfers to the elderly than to other age groups, and that the fraction of aid given in cash to families with children is relatively small (Currie and Gahvari 2008). Hidden saving and associated moral hazard problems are less of an issue for the elderly than for other demographic groups. An alternative explanation is that the government gives only little cash to families with children because it is afraid of the parents spending the money on the wrong items and, thereby, neglecting the benefit of their children. Note that this alternative interpretation is compatible with our model, in that the inefficient behaviors of the parents are qualitatively equal to the hidden savings problem.

In this paper, we have offered a simple two-period economy with two types of transfers and two types of agents. The unemployed agent has a moral hazard problem—associated with informational asymmetry on the effort level—and a hidden saving problem—driven by the ability to save cash transfers. We characterize the incentive-constrained efficient allocation, in which the marginal rates of substitution between cash and in-kind transfers differ across the population. Since investigations of this paper suggest that over providing in-kind transfers improves economic efficiency by reducing moral hazard in job search, we have offered an explanation that expands the Samaritan’s dilemma and the tax efficiency arguments.

REFERENCES


\[27\] In the working paper version of this paper, we show that a proper mixture of taxation and subsidization exists to decentralize the constrained efficient allocation.


A. APPENDIX

A.1. FIRST-ORDER CONDITIONS FOR THE PLANNER’S PROBLEM IN (7)

The first order conditions are:

\[
x_o : \beta u'(x_o) + \mu \lambda_H u''(x_o) + \mu \lambda_L u''(x_o - \sigma_L) + \mu \alpha [u'(x_o) - u'(x_o - \sigma_L)] = \mu p_x
\] (A1)

\[
X_o(s) : \beta U'(X_o(s))P(s|e_H) - \mu \lambda_H U''(X_o(s))P(s|e_H)
- \mu \lambda_L U''(X_o(s) + \sigma_L)P(s|e_L) + \mu \alpha [U'(X_o(s))P(s|e_H)
- U'(X_o(s) + \sigma_L)P(s|e_L)] = \mu p_x P(s|e_H)
\] (A2)

\[
y_o : \beta v'(y_o) = \mu p_x
\] (A3)

\[
Y_o(s) : \beta V'(Y_o(s))P(s|e_H) = \mu p_x P(s|e_H) + \mu \alpha V'(Y_o(s))P(s|e_L) - P(s|e_H)
\] (A4)

\[
x_j : (1 - \beta)u'(x_j) = (1 - \mu)p_x
\] (A5)

\[
X_j(s) : (1 - \beta)U'(X_j(s))Q(s) = (1 - \mu)p_x Q(s)
\] (A6)

\[
y_j : (1 - \beta)v'(y_j) = (1 - \mu)p_y
\] (A7)

\[
Y_j(s) : (1 - \beta)V'(Y_j(s))Q(s) = (1 - \mu)p_y Q(s)
\] (A8)

A.2. PROOFS AND DERIVATION

Derivation of (8): It is readily seen from (A5) and (A6) that \( u'(x_j) = U'(X_j(s)) \).
Similarly, (A7) and (A8) imply \( v'(y_j) = V'(Y_j(s)) \). Equations (A1) and (A2) imply \( u'(x_o) \neq U'(X_o(s)) \) unless all the multipliers—\( \lambda_H, \lambda_L, \) and \( \alpha \)—are zero. In Proposition 1, however, we show that \( \alpha > 0. \) (The result of Proposition 1 does not depend on \( u'(x_o) \neq U'(X_o(s)) \)). Substitute (A3) into (A4), and note that \( v'(y_o) \neq V'(Y_o(s)) \) as long as \( P(s|e_H) \neq P(s|e_L) \).

Proof of Proposition 1: We first prove that the multipliers for Euler equations (2) and (3) are zero.

Lemma 1. \( \lambda_H = \lambda_L = 0. \)

Proof. Summing up (A2) over \( s \in \{f, n\} \), we derive

\[
\beta \sum_s U'(X_o(s))P(s|e_H) - \mu \lambda_H \sum_s U''(X_o(s))P(s|e_H) - \mu \lambda_L \sum_s U''(X_o(s) + \sigma_L)P(s|e_L)
+ \mu \alpha \sum_s U'(X_o(s))P(s|e_H) - \sum_s U'(X_o(s) + \sigma_L)P(s|e_L)] = \mu p_x \sum_s P(s|e_H)
\]
Applying (2), (3), and \( \sum_t P(s|e_H) = 1 \), we obtain
\[
\beta u'(x_o) - \mu \lambda_H \sum_s U''(X_o(s)) P(s|e_H) - \mu \lambda_L \sum_s U''(X_o(s) + \sigma_L) P(s|e_L) \\
+ \mu \alpha [u'(x_o) - u'(x_o - \sigma_L)] = \mu p_x
\]
Comparing the above equation with (A1), we conclude that \( \lambda_H = 0 \) and \( \lambda_L = 0 \) since both \( u''(\cdot) \) and \( U''(\cdot) \) are negative.

Substituting Lemma 1 into (A1) and (A2), we derive (9) and (10).

**Comment:** A positive multiplier typically implies a binding constraint. A binding constraint, however, does not necessarily imply a positive multiplier. Our model is a special case in which a binding constraint has a zero multiplier. A similar phenomenon occurs in Chien and Song (2013) and Ábrahám et al. (2011). Intuitively, zero shadow values do not mean that saving constraints are meaningless. For instance, the planner would want to change \( \sigma \) if she could, and this change would eventually reduce the agent’s utility. However, this utility reduction would be made possible by either the increase or the decrease in \( \sigma \). The effects of an increase or decrease have identical magnitude, but in the opposite directions. Thus, shadow values become zero, although the constraints are still binding.

In addition, it is readily seen from (10) that \( \alpha \) and \( p_x \) cannot be zero at the same time. Otherwise, (10) becomes \( 0 = \frac{\beta}{\mu} \) — a contradiction. Next, suppose \( \alpha = 0 \), implying that \( X_o(s) \) is a constant. This means that transfer payments from government does not depend on state \( s \). Also note that \( Y_o(s) \) does not depend on \( s \) if \( \alpha = 0 \). Then the unemployed agent does not have any incentive to make high effort—a contradiction. Finally, suppose \( p_x = 0 \). Since this implies that the shadow value of \( x \) is zero, extra resource does not increase welfare. However, if we channel extra resource to the employed (who does not have the moral hazard problem), the welfare clearly increases.

**Proof of Proposition 2:** Suppose \( \sigma_L \leq 0 \). Then, (9) and (10) imply:
\[
\frac{p_x}{u'(x_o)} = \frac{\beta}{\mu} + \alpha \left( 1 - \frac{u'(x_o - \sigma_L)}{u'(x_o)} \right) \geq \frac{\beta}{\mu} \geq \mathbb{E} \left( \frac{p_x}{U'(X_o(s))} \right)
\]
since \( u'(x_o) \geq u'(x_o - \sigma_L) \) and \( U'(X_o(s)) \leq U'(X_o(s) + \sigma_L) \).

Then, by Jensen’s inequality,
\[
\frac{p_x}{u'(x_o)} \geq \mathbb{E} \left( \frac{p_x}{U'(X_o(s))} \right) > \frac{p_x}{\mathbb{E}(U'(X_o(s)))} \Rightarrow u'(x_o) < \mathbb{E}(U'(X_o(s))).
\]
Note that the strict inequality in the last term comes from the property that $U(\cdot)$ is strictly concave. Because this contradicts constraint (2), we conclude that $\sigma_L > 0$.

**Proof of Proposition 3:** Dividing (A1) by (A3) and dividing (A5) by (A7), and using Lemma 1, we derive:

$$\frac{u'(x_o)}{v'(y_o)} \left[ 1 + \frac{\mu}{\beta} \alpha \left( 1 - \frac{u'(x_o - \sigma_L)}{u'(x_o)} \right) \right] = \frac{p_x}{p_y} = \frac{u'(x_j)}{v'(y_j)}.$$

Since $\sigma_L > 0$ from Proposition 2, the result in (12) follows.