The Rise and Fall of Miracles

Jang-Ok Cho and Sookyoung Kim

Abstract It is shown that an economy can grow endogenously in early stage of development. However, the growth is not due to the factors emphasized in the literature but due to abundant labor. If labor is abundant enough to render real wage rate fixed as Lewis (1954) postulated, the marginal product of capital does not decrease with capital and hence endogenous growth emerges. However, the endogenous growth is temporary in the sense that once labor has been fully utilized, the growth enters neoclassical phase in which the economy converges along saddle path to its steady state of low growth. The model is proposed to explain the difference between West German and Japanese growth pattern after World War II. It is argued that West Germany was a highly industrialized Solow (1956) economy far below her steady state due to wartime destruction. As was predicted by Solow, West German growth rate was extremely high initially. However, her growth pace slowed down gradually from the beginning to a low growth steady state. By contrast, Japanese economy after the war was a largely agrarian and labor abundant Lewis economy. Japan’s rapid economic growth which had been sustained temporarily for about twenty years before she took the path converging to the present steady state of low growth was endogenous. Almost all of the growth miracles since the latter half of the twentieth century have been of Japanese pattern.

Keywords Macroeconomic Analyses of Economic Development; Industrialization; Manufacturing and Service Industries; Choice of Technology

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Labour is prior to, and independent of capital. Capital is only the
fruit of labour, and would never have existed. Labour is the superior
to capital, and deserves much higher consideration.
– Abraham Lincoln

1. INTRODUCTION

hough it cannot be denied that war is a paradigm of the dark side of human
nature, anybody of sound mind will not praise a war, not to speak of a war like
World War II. Looking intently at the numbers regarding casualties and damages
during the war, we would like to deny from our deep heart the notion that history
repeats itself. Although the tally of World War II casualties and damages has
been forgotten by most citizens of the world, our tapering memory of the war
should not keep us from keeping the lessons in mind.

After all the destruction and atrocities, a war entails unfortunate but in-
evitable economic experiment. Although it depends on the extent of wartime
destruction, reconstruction after the war may create a new economy quite differ-
ent from the prewar one. World War II was not an exception. Considering the
extent of tragic destruction of working age population and of production facili-
ties, housing, transport system and other properties, we can say that the recon-
struction after the Second World War was indeed the most serious and important
economic experiment in human history.

Amazing economic successes of both victorious and defeated nations un-
folded after the war. The victorious countries like U.S., U.K. and France enjoyed
rapid economic growth at least up to the first oil shock in 1973. The defeated
countries like West Germany, Italy and Japan experienced even more rapid eco-
nomic growth. Especially the West German and Japanese successes were dubbed
miracles. The miracles were possible most importantly by market oriented re-
form and introduction of new institutions facilitating competition, which were
forcefully imposed by the occupied forces, especially U.S.2 Obviously massive
investment and well trained workforce were another impetus for the miracles.

Although the West German and Japanese economic successes after World
War II could be categorically dubbed miracles, their growth paths differed sig-
ificantly from each other. The rate of West German income growth was exception-
ally high initially after the war. However, the pace slowed down from the

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1 From the message to the congress on December 3, 1861.
2 See Hirshleifer (1963) for Japan, pp.73-75, and for West Germany, pp.107-110.
start gradually to her steady state of low growth. The growth path followed by West Germany after the war was neoclassical. By contrast, an extremely high average growth rate had been sustained in Japan for about twenty years before it started to decline as early as 1971 gradually to her present steady state of slow growth. The Japanese growth for about twenty years after the war which did not have a tendency to converge to a steady state was not neoclassical.

Since mid 1980s, there have been seminal innovations in the paradigm of economic growth. The theories of endogenous growth pioneered by Romer (1986, 1990), Lucas (1988), Rebelo (1991), Grossman and Helpman (1991a, 1991b), and Aghion and Howitt (1992) have been developed to explain the growth experiences of advanced economies without declining trend. The Japanese growth for about twenty years after the war was endogenous in the sense that the average growth rate was sustained high without a declining tendency.

However, the endogenous growth theories have difficulty in explaining Japanese experience. Firstly, it was temporary only for about twenty years. Secondly, it was hardly plausible that the growth factors like knowledge externality, human capital investment, learning by doing, R&D investment etc. were temporarily large and grew fast enough to bring about such a high growth for a limited period of time. Furthermore, if the high growth sustained temporarily was caused by the factors, what changes in them caused Japan to leave all of sudden the sustained high growth path to take the convergent path to the present steady state of uncharacteristically low growth?

West German economy was far below its steady state due to the destruction of production facilities, urban housing and infrastructures during the war. The physical destruction resulting from the air attack on Japan approximated that suffered by Germany. Although wartime casualties and the extent of damage to productive capital were certainly not the same, there was no such a critical difference in wartime destruction between the two economies as to render the postwar growth paths so distinct from each other. In fact, the key difference after the war between the two economies was elsewhere. Furthermore, it was

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3The average growth rate of real GDP per capita between 1951 and 1970 was 7.9 percent. For twenty years, Japanese real GDP per capita increased by a factor of 4.6.
4See Barro and Sala-i-Martin (2003) for an illuminating survey.
5The transitional dynamics in two sector models of endogenous growth probed in depth by Mulligan and Sala-i-Martin (1993) is also far from Japanese experience. See their figures I (p.762) and II (p.767) from simulated economies.
6See the United States Strategic Bombing Survey (1987), p.86. See also the United States Strategic Bombing Survey (1946a), pp.3-23 for the details of casualties and damages inflicted by the attacks.
7See the next section for details.
remotely related to the wartime destruction itself.

The proportion of primary sector employment after the war differed enormously between the two economies. West Germany’s employment in primary sector was about 24.6 percent in 1950. The remaining 75.4% of employment was split between the secondary (42.9%) and tertiary industry (32.5%). Japan’s employment in primary sector in 1950 was about 48.5 percent. The remaining 51.5% of employment was split between the secondary (21.8%) and tertiary industry (29.6%). In sum, Japan was largely agrarian, whereas West Germany was highly industrialized.

The composition of labor and capital across sectors is believed to be the key to understanding the distinct growth patterns after the war. We postulate that abundant labor in Japan instigated and sustained unusually high growth after the war, temporarily though, before transition toward her steady state of low growth and then ask what theory can explain the pattern. It is argued that Japanese growth pattern after the war is well in line with celebrated Lewis (1954) model.

Abundant labor is indeed the key factor in Lewis model of economic development. His model has two sectors, namely agriculture (primary) and industry (secondary and tertiary). Labor is abundant and employed mostly in primary sector. Primary sector real wage is fixed at subsistence level due to abundant labor. Industry can hire labor from primary sector and expand production without increase in unit labor cost, which brings about industrial development.

Although we do not have two sectors in this paper, labor is assumed to be abundant. If real wage rate is kept constant due to abundant labor in an economy equipped with technology homogenous of degree one, the marginal product of capital is also constant and hence $AK$ technology emerges in equilibrium. That is, growth takes place endogenously. However, as soon as labor has been fully utilized and real wage rate started to be associated and increase with capital, the endogenous growth ends and neoclassical convergence to a low growth steady state sets in. Now the pattern of Japanese miracle emerges.

Note once again that the factor causing the temporary endogenous growth is not the ones extensively debated in the endogenous growth literature but abundant labor. However, this does not mean at all that the endogenous factors are not important. Indeed they are among the key determinants of the rate of high speed growth. Furthermore, after the transition to a steady state has been com-

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8The proportion of rural population in 1950 was 28.9 percent in West Germany and 62.5 percent in Japan.
9See Fei and Ranis (1964) for an extension with rising real wage schedule in primary sector. Ranis (2004) summarizes the contribution made by Lewis.
10In fact, wage rate needs not be constant for endogenous growth. See section 5 for the intuition.
pleted, the growth rate of GDP per capita is not only independent of labor but also determined by the growth factors either endogenous or exogenous.

The paper consists of six sections. In the next section, we will look at the growth patterns of West Germany and Japan after World War II and discuss possible factors which caused the difference. The model is presented in the third section and equilibrium technology is analyzed in the fourth section. The fifth section discusses the issues related to the model and the sixth section concludes.

2. WHAT WAS DIFFERENT?

The time paths after World War II of the growth rate of GDP per capita of the two miracles, West Germany and Japan, are depicted in figure 1. After the tumultuous postwar period between 1945 and 1950, West Germany followed a typical growth path predicted by Solow (1956, 1957). Starting from below her steady state with an exceptionally high rate of growth, West Germany converged gradually to the steady state of low growth. The five year averages of West German growth rates were 8.6, 6.0, 3.7, 3.5 percent between 1951 and 1970, and 4.1 percent between 1971 and 1973. West Germany seemed to arrive at a steady state as early as 1980s.

After the war, Japan had first achieved extremely high economic growth without declining trend for about twenty years before starting to converge gradually to her steady state of low growth. The five year averages of Japan’s growth rates of GDP per capita were 7.6, 7.6, 8.3, 10.4 percent between 1951 and 1970, and 5.6 percent between 1971 and 1973. Japan had taken declining growth path since early 1970s and finally arrived at her steady state of low growth as early as 1992. Then so-called Japan’s lost decades set in.

To differentiate between the two miracles, we ask what differed between them after the war and made the growth paths distinct thereafter. First, we look into the extent of the wartime destruction of physical capital. Second, we tabulate wartime casualties. Third, we compare employment composition across industries and the extent of industrialization.

(i) Capital

Most destruction of German capital stock during the war was brought about by aerial attacks. Allied forces initially concentrated on military targets. In the

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11 Business cycle fluctuations are prominent in the figures. However, it is fortunate that the long run trend can be easily identified even with the naked eye.
12 See Hayashi and Prescott (2002) for the causes of the lost decade.
spring of 1943, allied naval and air power scored a definite victory over German submarines. By the spring of 1944 opposition of the German airforce (the Luftwaffe) had ceased to be effective and on D-day of Normandy invasion the Luftwaffe had only 80 operational planes with which to oppose the invasion.

In addition to military targets, a large number of civilian industries were hit heavily. However, the most significant attack was on transportation and urban housing. The attack on transportation including rail and inland waterways was the decisive blow that completely disorganized the German economy and produced a serious disruption in traffic over all of western Germany.

It was estimated that 490,000 residential buildings were destroyed and 415,000 heavily damaged, which represented 20 percent of all dwelling units in Germany and about 50 percent of the dwelling units in the cities subjected to major air attack. The result of the destruction was to render homeless some 7,500,000 German civilians. The bottleneck due to the lack of urban housing and damaged transport system was severe obstacle to economic recovery after the war. Many West Germans and the majority of immigrants from Eastern Europe had to reside in rural area, which was far from the industrial production center.

The overall level of Japanese war production began to decline due to the interdiction of overseas imports. The increasing stringency of shipping routes reduced Japan’s coal and iron ore imports by two thirds by the middle of 1944. The steel shortage constituted an overall limitation on the war potential of the Japanese economy. Oil was of critical importance to Japan’s military machine and to her merchant marine. Oil imports from the south began declining in August, 1943, and was eliminated completely by April, 1945 after the liberation of the Phillipines and the capture of Okinawa.

Japan was critically wounded by military defeat in Pacific theater, destruction of most of her merchant fleet and almost complete sea blockade. Since November 1944, US long range bombing offensive from the Marianas such as Guam, Saipan and Tinian began. The aerial attack concentrated initially on aircraft factories, arsenals, electronics plants, oil refineries, and finished military

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13 See The United States Strategic Bombing Survey (1945a), pp.68-70.
14 See ibid., pp.11-29.
15 It was June 6, 1944.
16 Commercial highway transportation of freight was insignificant. It accounted for less than three percent of the total.
17 See ibid., pp.59-64.
18 See ibid., pp.136-137.
19 More than 80% of ships were destroyed or went down at sea.
goods etc. Urban area attacks were initiated in March, 1945, which destroyed much of urban housing and transport system.

The tonnage dropped on Japanese territory was only 12 percent of that on German soil. However, the attack on Japan was more concentrated in time and the target areas were smaller and more vulnerable. In sum, the physical destruction resulting from the air attack on Japan approximated that suffered by Germany. About 25 percent of Japanese wealth was destroyed by US aerial attacks. If the losses of purely military assets were included, the damage rose to 41.5 percent of total wealth. The destruction of industrial capital amounted to 34.3 percent. 25 percent of buildings was destroyed and one third of urban housing burnt down.

(ii) Labor

Wartime casualties differed between West Germany and Japan. Table 1 shows the casualties during the war in belligerent nations. The casualties in World War II were beyond our imagination. German population during the war was about 78 million. About 17.9 million among them served in the military forces, of whom 3.25 million were killed or missing during the war and 4.66 million were wounded. German civilian casualties including those killed, missing or wounded were more than 4 million. In sum, German casualties were well over 10 million.

On the other hand, there was a massive influx of immigrants after the war into West Germany (the British and American zones). Between 1938 and 1946, West German population rose from 34 million to 38.9 million, an increase of 4.9 million. 6.7 million (17.5%) among the population were immigrants including migrants, refugees and expellees mostly from Soviet occupied Eastern Europe to avoid ethnic cleansing. The immigrants were highly skilled.

Japanese population was about 72.2 million, of whom 9.1 million served in the military forces. 1.74 million among them were killed or missing, 0.94 million wounded and 0.41 million were captured as P.O.W. Civilian casualties

20The attack was mostly preparing for invasion of Japanese home islands planned in November, 1945.
21Incendiaries were used instead of high explosive bombs.
22The total tonnage of bombs dropped by allied planes on the home islands of Japan was 160,800 tons, while the total tonnage dropped on German soil was 1,360,000 tons. See The United States Strategic Bombing Survey (1987), P.86.
24See table 51 in Ellis (1995) for other countries in the war.
26See also table 2.1-3 in ibid., p20.
were about 0.67 million. In sum, more than 3.3 million soldiers and civilians were killed, missing or wounded.

Japan also had a large influx of immigrants after the war. Most of the Japanese living outside of Japan, mostly in the previously occupied area, returned to home islands due to either demobilization of troops or repatriation. Japanese population and labor force were also larger in a few years after the war ended than before and during the war. The population in 1947 was 78 million, which was 6.9 percent larger than in 1944. The number of employed persons in 1947 was 33.3 million, which was 4.8% larger than in 1944.

Summing up the facts regarding the West German and Japanese economy immediately after World War II, we have finally arrived at the following picture. Capital was substantially destroyed overall in both countries. The wartime destruction of capital was not critically different enough to make growth paths distinct for the two economies. Although both West Germany and Japan had horrendous casualties, population and labor force increased substantially in a few years after the war due to immigration and were not a cause for the differing growth experience thereafter.

(iii) Sectoral Employment and Industrialization

The key difference between the two economies after World War II can be found elsewhere. Furthermore, it was remotely related to the war and wartime destruction. The industrial composition of employment differed markedly between the two economies. West German employment in primary sector in 1950 was 24.6% of the employment. The remaining 75.5% of employment in the year was split between the secondary (42.9%) and the tertiary (32.5%) industry.

On the other hand, Japanese employment in primary sector in the same year was officially 48.5% of aggregate employment. The remaining 51.5% of employment in the year was split between the secondary (21.8%) and the tertiary (29.6%) industry. Figure 2 shows the trend of primary sector employment in West Germany and Japan since 1953. The proportion of primary sector employment has gradually decreased in the two economies. Japan’s primary sector employment reached the rate comparable to that of West Germany in 1950 fourteen years later in 1964.

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27 See Kosai (1986), table 2-5, p.40.
28 See the official statistics at the internet site of German Federal Statistical Office. Also see Ifo Institute for Economic Research and Sakura Institute of Research (1997), pp.118-119.
29 See ibid., p.108. The employment statistics before 1950 can be found in Nakamura (1995), p.15.
30 In six years, Japan took the Solow path of convergence to the slow growth steady state. This transition from sustained to decelerating growth was sudden and started three years earlier before
The urban population in the two countries after the war can be found in table 2. Due to the massive destruction of urban housing during the war, it declined in both countries right after the war. However, it bounced back to the prewar level by 1950. The urban (rural) population of West Germany in September, 1950 was 71.1 (28.9) percent and that of Japan was 37.5 (62.5) percent. Compared with primary sector employment, the proportion of rural population was disproportionately larger in Japan.

In sum, West Germany was much more industrialized than Japan by the time World War II ended. It is argued that the distinct growth patterns after the Second World War were due to the differed extent of industrialization and labor abundance. With the benefit of hindsight, we can categorize West Germany as a Solow (1956) economy not only with highly industrialized technology but also with capital/labor ratio lower due to wartime destruction. By contrast, Japan can be categorized as a Lewis (1954) economy not only less industrialized but also with reserve army of workers in the primary sector.

When capital/labor ratio is lower than in steady state, the growth path in a Solow economy is well known. However, that in a Lewis economy has not been probed very much in growth literature. If labor is abundant enough to render real wage rate fixed, it is argued that the economy grows endogenously until labor is fully employed.

3. THE ECONOMY

We assume many identical agents who live forever. Each agent is endowed with fixed amount of time each period. Following Hanoch (1980), Bils and Cho (1994), and Cho and Cooley (1994), we introduce a few time dimensions of labor supply. Each agent is endowed with fixed number of weeks and with fixed

\[ \text{the first oil shock.} \]

\[ 31 \text{Japanese industry expanded rapidly in 1930s. Industrial employment had been 5.8 million in 1930 and increased to 8.1 million in 1940. It increased by about 40 percent in a decade. However, agricultural employment declined from 14.1 million by only 0.5 million. The additional labor force almost entirely was provided for by the growth in population. See United States Strategic Bombing Survey(1946b), pp.13-14.} \]

\[ 32 \text{United States Strategic Bombing Survey (1946b) concluded that although Japan was rapidly industrialized in 1930s, it remained until the outbreak of war "essentially agrarian economy in which roughly half of the population was engaged in feeding the nation." Furthermore, there was a considerable manpower "cushion" consisting of "hidden unemployed". See p.14.} \]

\[ 33 \text{How much employment is the fully employed labor in this context has to be defined empirically. See section 5 for a discussion.} \]
amount of time in each week, which can be devoted to market work or leisure. The fraction of weeks devoted to market work is denoted by $e_t$, which is assumed to be continuous and constrained between 0 and 1. Working hours in a week are indivisible and denoted as $n$ without time subscript.

The representative agent now chooses the weeks worked together with consumption and investment. Each agent maximizes his lifetime utility.

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, n, e_t),$$

where $c_t$ is consumption and $\beta$ is the utility discounting factor. We assume the following temporal utility function.

$$u(c_t, n, e_t) = \frac{1}{1-\sigma} \left( c_t - \frac{B}{1+\gamma} n^{1+\gamma} e_t \right)^{1-\sigma}, \quad \sigma, \ B, \ \gamma \geq 0$$

Here the disutility of working $n$ hours in a week is:

$$\frac{B}{1+\gamma} n^{1+\gamma},$$

which is summed over the weeks worked, $e_t$. We let $\frac{B}{1+\gamma} n^{1+\gamma} = B_1$. If we assume the fraction of weeks worked by the agents is distributed evenly across a period\footnote{Consider an example. Suppose there are three weeks and three workers, the optimal number of weeks are two and optimal working hours are forty. The first worker works forty hours each in the first and second week. The second worker works forty hours each in the second and third week. The third worker works forty hours each in the first and third week. Then employment rate is 66.6\%(2/3). Hence the fraction of weeks worked is the employment rate. Of course, we treat all weeks the same and abstract from the holiday and seasonal effect.}, we can interpret $e_t$ as employment rate.

The aggregate hours of work in efficiency unit in a period is $h_t n e_t L_t$, where $h_t$ and $L_t$ denote the growth factor (for example, human capital) and labor force. The aggregate production technology is Cobb-Douglas.

$$Y_t = K_t^\theta (h_t n e_t L_t)^{1-\theta}, \quad 0 < \theta < 1,$$

where $K_t$ denotes aggregate capital stock. The initial values for $K_0$ and $h_0$ are given as $K_0$ and $h_0$. We will denote an aggregate variable by a capital letter and its individual counterpart by the corresponding lowercase letter. By dividing both sides of \footnote{Consider an example. Suppose there are three weeks and three workers, the optimal number of weeks are two and optimal working hours are forty. The first worker works forty hours each in the first and second week. The second worker works forty hours each in the second and third week. The third worker works forty hours each in the first and third week. Then employment rate is 66.6\%(2/3). Hence the fraction of weeks worked is the employment rate. Of course, we treat all weeks the same and abstract from the holiday and seasonal effect.} by $L_t$, we have the per capita production.

$$y_t = k_t^\theta (h_t n e_t)^{1-\theta}, \quad y_t = \frac{Y_t}{L_t}, \quad k_t = \frac{K_t}{L_t}$$
The per capita resource constraint is the following:

\[ c_t + i_t = y_t, \]

where \( i_t \) is per capita investment. The law of motion of capital stock is the usual one.

\[ k_{t+1} = (1 - \delta)k_t + i_t, \]

where \( \delta \) is the capital depreciation rate and the time to build is one. Labor force and the growth factor are assumed to grow at fixed rates.\(^{35}\)

Now the competitive equilibrium can be obtained by solving the following programming problem:

\[
V(k_t, h_t) = \max_{c_t, i_t, h_t} \frac{1}{1-\sigma} (c_t - B_1 e_t)^{1-\sigma} + \beta V(k_{t+1}, h_{t+1})
\]

s.t.

1. \( c_t + i_t = k_t^{\theta}(h_t e_t)^{1-\theta} \)
2. \( k_{t+1} = (1 - \delta)k_t + i_t \)
3. \( h_{t+1} = (1 + g)h_t \)
4. \( 0 \leq e_t \leq 1 \)
5. \( k_0, h_0 \) is given.

where \( V(\cdot, \cdot) \) is the value function and \( g \) is the growth rate of the growth factor. This is a standard growth problem except that labor decision involves only extensive margin \( (e_t) \).

4. EQUILIBRIUM TECHNOLOGIES

We assume in this section that the labor force and the growth factor are fixed at \( L \) and \( h \). Then we have the following control rules.\(^{36}\)

\[
\begin{align*}
  y_t &= \left[ \frac{(1-\theta)h_t}{B_{1t}} \right] \frac{1}{\sigma} k_t \\
  e_t &= \left( \frac{1-\theta}{B_{1t}} \right)^{\frac{1}{\sigma}} (hn)^{1-\theta} k_t \\
\end{align*}
\]

if \( k_t < \left( \frac{1-\theta}{B_{1t}} \right)^{-\frac{1}{\sigma}} (hn)^{-\frac{1-\theta}{\sigma}} \)

\[
\begin{align*}
  y_t &= (hn)^{1-\theta} k_t^{\theta} \\
  e_t &= 1 \\
\end{align*}
\]

if \( k_t \geq \left( \frac{1-\theta}{B_{1t}} \right)^{-\frac{1}{\sigma}} (hn)^{-\frac{1-\theta}{\sigma}} \)

\(^{35}\)Making fertility and the growth factor endogenous does not alter the basic result in the paper. Since these important choices have been analyzed extensively in the literature, we will abstract from their endogeneity.

\(^{36}\)See the appendix 1 for derivation.
In the first set of rules, $0 < e_t < 1$, and hence labor is abundant in the sense that it is not fully employed. Note that the marginal product of capital is constant in the first set of rules in (5). Hence $AK$ technology emerges as the equilibrium technology and an endogenous growth takes place. However, the endogenous growth is temporary in the sense that it is effective only when labor is abundant. If the economy accumulates enough capital and reaches full employment, i.e. $e_t = 1$, the second set of rules in (5) is of equilibrium. The marginal product of capital in this case is diminishing and hence the economy converges to a steady state.

If capital depreciates fully in a period, i.e. $\delta = 1$, and $0 < e_t < 1$, the equilibrium consumption and investment rules can be obtained as:

$$c_t = (1 - \mu_t)y_t, \quad k_{t+1} = \mu_1y_t,$$

where $\mu$ is defined as:

$$\mu_1 = \left(\beta \theta\right) \frac{1}{\sigma} \left[\frac{(1 - \theta)hn}{B_1}\right]^{\frac{1 - \sigma}{\theta \sigma}}.$$

Hence the growth rate in the endogenous phase is:

$$\frac{k_{t+1}}{k_t} = \left(\beta \theta\right) \frac{1}{\sigma} \left[\frac{(1 - \theta)hn}{B_1}\right]^{\frac{1 - \sigma}{\theta \sigma}},$$

which is larger, the larger $\beta$ and $h$ are. The size of population (or labor force) does not affect the per capita growth rate itself but the initial capital stock per capita. If the population is larger given total amount of capital, the initial capital stock per capita is smaller and hence the endogenous growth phase will be prolonged.

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37In both cases, if $k_t = \left(1 - \theta \frac{hn}{B_1}\right)^{\frac{1}{\theta}}(hn)^{\frac{1}{\theta \sigma}}$, then $y_t = \frac{B_1}{1 - \theta}$. Of course, the transversality condition is satisfied.

38See the appendix 1 for derivation.

39However, if $e_t = 1$, we cannot solve for them analytically even with capital depreciating fully in a period. See the appendix 1 for a discussion.

40Although we hold $h$ fixed in the analysis, growing $h$ either exogenously or endogenously do not affect the result qualitatively. However, any changes $h$ affect the growth rate in the endogenous growth phase and hence have scale effect. See section 5 for a discussion.
5. DISCUSSION

Apart from West Germany and Japan, there have emerged many economic miracles since the latter part of the twentieth century. The Japanese pattern of growth has been prevalent among miracle economies. However, the extent and span of the miracles vary considerably. There are other issues related to the model and data.

(i) Intuition

With cursory treatment of preferences, we can have a simple intuition. Suppose an economy has a Cobb-Douglas technology.

\[ Y_t = AK_t^\theta N_t^{1-\theta}, \]

where \( N_t \) denotes aggregate labor. In addition, suppose labor is abundant and hence real wage rate is fixed at \( w \) as in Lewis (1954). The cost minimizing firm will equate the marginal cost of an input and its marginal product.

\[ w = (1 - \theta)AK_t^\theta N_t^{-\theta}, \quad r_t = \theta AK_t^{\theta-1}N_t^{1-\theta}, \]

where \( r_t \) is the real rental price of capital. We can have the following from the cost minimizing condition.

\[ \frac{K_t}{N_t} = \left[ \frac{w}{(1-\theta)A} \right]^{\frac{1}{\theta}}. \]

As far as real wage rate is fixed, equilibrium capital/labor ratio is constant. Using the ratio in the aggregate production function, we have the equilibrium technology.

\[ Y_t = \left( \frac{1-\theta}{w} \right)^{\frac{1}{\theta}} A^\theta K_t \text{ if } K_t < \left[ \frac{w}{(1-\theta)A} \right]^{\frac{1}{\theta}} N_t, \]

where \( N \) is the labor endowment. Hence an \( AK \) technology emerges in equilibrium.

In fact, real wage rate needs not be constant for endogenous growth. As far as real wage rate is dissociated from capital accumulation, labor may cause endogenous growth. For example, suppose there is a growth factor determined either exogenously like technological progress in Solow (1957) or endogenously like human capital accumulation in Lucas (1988). Then the aggregate production function can be written as:

\[ Y_t = K_t^\theta (h_t N_t)^{1-\theta}, \]
where $h_t$ is the growth factor. Now assume that real wage rate is proportional to the growth factor such as $w_t = wh_t^{1-\theta}$, where $w$ is fixed. In equilibrium, the following AK technology emerges:\footnote{See appendix 2 for derivation.}

$$Y_t = \left(\frac{1-\theta}{w}\right)^{\frac{1-\theta}{\sigma}} h_t^{1-\theta} K_t$$

Hence, endogenous growth emerges even with increasing wage.

Furthermore, note that there can be other causes of fixed wage or wage being dissociated from capital. For example, wage may be institutionally fixed or dissociated from capital. Policies like investment tax credit and/or subsidy may also help real wage be dissociated from capital accumulation.

(ii) Preference Transition

We have a preference transition in the model. The equilibrium preferences differ between developing and advanced stage of growth. For example, $\delta = 1$.

Then we can have the following when $0 < e_t < 1$:

$$c_t - B_1 e_t = \left(\frac{\mu_1 + \theta - 1}{\mu_1}\right) c_t$$

Hence the period utility in equilibrium is:

$$u(c_t, e_t) = \frac{1}{1-\sigma} (c_t - B_1 e_t)^{1-\sigma} = \frac{1}{1-\sigma} \left(\frac{\mu_1 + \theta - 1}{\mu_1}\right)^{1-\sigma} c_t^{1-\sigma}.$$  

However, once labor is fully utilized, i.e. $e_t = 1$, the period utility in equilibrium is:

$$u(c_t, 1) = \frac{1}{1-\sigma} (c_t - B_1)^{1-\sigma}.$$  

The period utility is of CRRA(constant relative risk aversion) type in early stage of development and it is of HARA(hyperbolic absolute risk aversion) class in advanced stage of growth.

(iii) Growth Patterns after World War II

Three distinct growth patterns emerged among the belligerent nations after World War II. Apart from the West German and the Japanese pattern, the third growth pattern could be found in U.K. and U.S. Roughly speaking, there was neither increasing nor decreasing trend of growth in U.K. and U.S. economy.
(see figure 3). The two economies were fully industrialized\footnote{The primary sector employment of U.K. was 5.1\% in 1950 and the remaining 94.9\% was split between the secondary (44.9\%) and tertiary (50.0\%) sector. U.S. primary sector employment was 12.5\% and the remaining 87.1\% was split between the secondary (33.6\%) and tertiary (53.5\%) sector. See Maddison (1997), table 5.} and in their steady states\footnote{Note that the war was carried out mostly continental Europe and Pacific area west of Hawaii. Hence U.S. lost virtually no production facilities in the war. Although U.K. was bombarded by the Luftwaffe, the destruction of production facilities was fairly limited. In sum, U.S. and U.K. physical capital was not affected much in the war. Furthermore, compared to Germany and Japan, their wartime casualties were much lighter.}.

However, the growth in other belligerent economies like France, Italy and USSR\footnote{The sectoral composition of employment of Italy, Spain and USSR in 1950 was not much different from that of Japan. However, the sectoral composition of France was 28.3\%, 34.9\% and 36.8\% in 1950 respectively for the primary, secondary and tertiary sector, which were rather similar to West Germany’s. However, the secondary sector employment was ten percentage point lower in France than in Germany. See Maddison (1997).} was of Japanese pattern with varying extent. Although it did not take part in the war, Spain\footnote{Note that Spain had a civil war between July, 1936 and April, 1939.} also followed the Japanese pattern. These economies were not fully industrialized by the time World War II ended in the sense that substantial fraction of labor was still employed in the primary sector\footnote{The rate of change in the sectoral composition of employment can be found in Maddison (1982), table 5.11, p.117.}. Furthermore, large part of their production facilities was destroyed during the war.

We find the same pattern in East Asian economies (see figure 4). Especially, Korea and Taiwan followed the Japanese growth pattern. Korean GDP per capita grew annually at the average rate of 7.1\% for twenty six years between 1966 and 1991 and Taiwanese economy at the average rate of 6.9\% percent also for twenty six years between 1964 and 1989. We find the same pattern in Hong Kong and Singapore with lesser extent.

Figure 5 shows the growth pattern of recent miracle economies. China and Indonesia are economies with abundant labor. Since the inception of reform, China is growing without declining trend. Although the East Asian crisis took a heavy toll on Indonesian economy, her sustained growth is robust. Recent world financial crisis also took a heavy toll on the transition economies. However, they grew without declining trend up to the crisis.

(iv) The Span of High Speed Growth

The labor-induced endogenous growth is temporary. However, it is not of short run either since it lasts at least for a few decades. The span of labor-induced high growth depends on two categories of factors. First, the factors determin-
ing initial distance from full employment matter. The span will be longer with scarcer initial capital and/or larger population (labor force). Faster population growth due to either high fertility or immigration also implies longer span.

Second, the rate (speed) of growth is another determinant of the span. As is analyzed in section 4, the growth rate is affected by the parameters like $\beta$, $\theta$, $\sigma$ and $B_1$. However, note in (7) that the distance from full employment does not affect the rate of growth in the endogenous growth phase. Also note that the level of growth factor $h$ affects the growth rate. That is, there is a scale effect in endogenous phase of growth. As Jones (1995a, 1995b) forcefully argued against it with empirical evidence, the scale effect is implausible in the long run. Note, however, that the scale effect in the model is temporary and disappears in the long run. The growth rate in the steady state is determined solely by the rate of growth of the factor $h_t$, not by its level.

Empirically speaking, the span of high speed growth is less than thirty in most of the economies. It took about twenty years before Japan took the convergent path. It took less than twenty years for advanced economies like France and Italy after World War II and about twenty six years for Korea and Taiwan to arrive at the break. Hence we may say that the high endogenous growth induced by labor in early stage of development mostly ends in less than thirty years.

One note regarding the break from high speed growth to saddle path convergence is that it has taken place in most of the miracles when the primary sector employment has reached around twenty percent. In other words, the full utilization of labor can be empirically defined as the primary sector employment of around twenty percent.

(v) Growth Accounting

Paul Krugman (1994) prophesied in a provocative article that the East Asian miracles would fall soon. In fact, the prophecy was realized in less than ten years. It was based on growth accounting performed by Kim and Lau (1994a, 1994b) and Young (1992, 1994, 1995). According to Kim and Lau (1994a), the East Asian miracles were made mostly by accumulating tangible factor inputs,

47 Easterly, Kremer, Pritchett and Summers (1993) obtained a timely and chillingly prophetic result that the high growth of East Asian Tigers was temporary and would soon decline. See also Easterly (1995).

48 One notable exception so far is China. China is enjoying exceptionally long span of high sustained growth. Taking 1978 as the starting year, the span is thirty six years (1978–2013), and, taking 1983 as the starting year, the span is thirty one years (1983–2013). However, China, by all means, seems to be approaching the break. See Li, Li, Wu and Xiang (2012) and Zhu (2012) for recent labor market in China.
i.e. capital and labor. By contrast, technical progress was the most important source of postwar economic growth of France, West Germany, U.K. and U.S. As for Japan, capital accumulation accounted for the largest portion, with technical progress a close second. Kim and Lau (1994b) added a measure of human capital in their regression to find no changes in the result.\[49\]

In a series of important contributions, Young (1992, 1994, 1995, 2003) cleverly constructed theoretically sensible data and found that the key to the East Asian miracles was the accumulation of production factors. He observed that

"... labor-deepening (the rise in participation rates, transfer of labor out of agriculture, and improvements in educational attainment) and not capital-deepening is the key force explaining the extraordinary improvement in per capita living standards, in the presence of moderate total factor productivity growth, achieved by the high growth economies of East Asia." (2003, pp.1258-1259)

Furthermore, he concluded that

"Despite the popular academic emphasis on industry and exports, a deeper understanding of the success of the world’s most rapidly growing economies may lie in that most fundamental of development topics: agriculture, land, and the peasant." (2003, p.1260)

The deconstruction of the miracles by Krugman, Kim and Lau, and Young is well in line with the model in sections 3 and 4. The growth of the miracle economies is attributable to abundant labor. Moreover, Young’s conclusion that labor rather than capital deepening was "the key force" is also in line with the finding in the sections.

One note is that as the phase of development evolves, the sources of growth are changing over time. Especially, those differ significantly between during and after miracles. Hence the growth during and after miracles has to be treated separately in regression based growth accounting as in Kim and Lau (1994a, 1994b).\[50\]

(vi) Convergence

\[49\] The sample period does not accord with the miracle years. It is from 1950 to 1990 for U.S and from 1958-1990 for France, West Germany, Japan and U.K. Hence the result is not very informative regarding French, West German and Japanese miracles.

\[50\] Although Young (1992, 1995) used a regression based growth accounting, his sample period covered the miracle years only.
Since the important contributions by Abramovitz (1986), Baumol (1986), Barro (1991, 1997), Barro and Sala-i-Martin (1992), and Mankiw, Romer and Weil (1992), income convergence among the nations has been one of the key concerns in empirical growth literature. The time path since 1870 of per capita GDP of sixteen advanced economies studied by Maddison (1982), Abramovitz, and Baumol[51] is depicted in figure 6. The choice of the countries for the study of convergence is rightly criticized for selection bias by De Long (1988). Notwithstanding, an important information regarding the role of miracles on convergence can be distilled from the figure.

The vertical axis in figure 6 denotes the relative GDP per capita to U.S. counterpart in percentage term. Figure 6(a) shows that the dispersion of GDP per capita among the sixteen economies has been shrinking, i.e. σ-convergence in Barro and Sala-i-Martin’s parlance has been taking place. However, disentangling the paths tells us a somewhat different story. The countries are divided into four groups according to GDP per capita in 1870 and then the path of GDP per capita relative to U.S. counterpart is depicted in figures 6(b)-(e).

In Switzerland and among the countries in Club B and Club C we cannot find any significant catch-up until 1950. Before 1950, the only catch-up was by U.S., which can be found in figure 6(b). In fact, U.S. forged ahead as the industrial leader as early as 1900 and the other leaders, Australia, Netherland, Belgium and finally U.K., fell behind U.S. since then. The convergence among Club A countries before 1950 was not by catching-up but by the early leaders’ falling behind. The convergence among the sixteen countries before 1950 in figure 6(a) reflects this falling behind, which is contrary to catch-up hypothesis and spurious in the sense.

As Abramovitz (1986), Abramovitz and David (1996), and Crafts (2004) noted, the only meaningful catching-up convergence among Maddison’s 16 took place between the end of World War II and 1980. After the turbulent era of the Great Depression and World War II[52] there emerged miracles of a pattern or another among the laggards. Figures 6(b)-(d) show that every country except Switzerland was catching up to U.S. after the war[53]. However, the catching-up convergence has stopped since early 1980’s except for a few outliers, Norway and Switzerland. The catching up convergence among Maddison’s 16 took place

---

[51] Their focus was on productivity convergence rather than income.
[52] Divergence took place during the wartime. However, it was obviously due to the temporary shocks. See figure 6(f).
[53] Note that the most impressive catching-up was by Club C countries and notably by Japan among them. Japan’s per capita income before the war was only 30% of US counterpart. However, it was more than 80% in 1990.
solely during the era of miracles. As soon as the miracles fell, catching up to the leader, U.S., has stopped.\footnote{In fact there has been a slight divergence among the Maddison’s 16 since 1990. See Crafts (2004) for related discussions.} In other words, miracles did virtually everything for the convergence among Maddison’s 16.

According to Jones (1997), economic miracles in the world are occurring more frequently\footnote{See table 1 in his paper.} than disasters. As can be seen in figures 4 and 5, almost all of the miracles presently going on are of Japanese pattern and hence their catching up will be rather rapid. Considering both miracles occurring more frequently and the growth potential intrinsic in labor, we may conclude that more and more economies are expected to join miracle club and hence convergence will be the long run trend.\footnote{The conclusion is in line with Lucas (2002), p175. See also Sala-i-Martin (2006) for an estimate of world income distribution, which is shrinking due to recent miracles in populous economies like China, India and Indonesia.}

(vii) Why After World War II?

Figure 6 shows that the convergence of GDP per capita has been a postwar phenomenon even among the advanced economies. U.S. had caught up with U.K. as early as 1900 but other than that there was no convergence before World War II in the sense of catch-up. Even conditional convergence did not take place, not to mention absolute convergence. The West German and the Japanese miracle after World War II and their convergence to U.S. were unprecedented. The rapid growth of France and Italy’s GDP per capita after the war was also unprecedented.

In retrospect, labor must have been abundant in most of the countries before the Second World War. However, the patterns of miracle growth can not be found before the war even in the advanced economies and obviously neither in developing countries. What was different between before and after World War II? Although we do not have a definite answer to the question at the moment, we can find many clues to it in Abramovitz (1986) and Abramovitz and David (1996).

Most of all, U.S. technologies since early nineteen century were natural resource-intensive, tangible capital-using, and scale-dependent for mass production and high throughput\footnote{Abramovitz and David (1996), p.25.}. They could not be readily imitated by the laggard countries with scarce natural resources and narrow market. However, the use of cheap petroleum and the development of low cost transport made the laggards’ resource constraint less and less severe and their market size was also expand-
ing rapidly. Furthermore, there were shifts in the direction of innovation that favored intangible assets such as human capital and R&D patents rather than tangible capital.

As is well known, development consists of long succession and interaction of not only economic but also political, social and cultural changes. Political, social and cultural attributes, qualities and characteristics of people, institutions,\textsuperscript{58} tradition and policies influence the response of people to economic opportunity,\textsuperscript{59} which have to do with the concept of social capability popularized by Abramovitz (1986).\textsuperscript{60} However, any rapid changes of social capability are not ordinary. In fact, it was not favorable for economic growth before the war. Abramovitz and David (1996) observed the following.

"The most important change of outlook was in the public attitude towards economic growth itself. In the first half of the century, and particularly in the interwar years, the major concerns had been income distribution, trade protection, and unemployment. After World War II, it was growth that gripped people’s imagination, and growth became the premier goal of public policy."(p.57)

War in many cases causes a paradigm shift. After the turbulent first half of the twentieth century, the stage for economic miracles was set. More technologies were available and the cost of their adoption was lower. Social capability matured finally after disastrous conflicts. The world economic and political order was designed much better than the prewar one. Better institutions governing international relation and trade were created. Free trade and free movement of capital across national borders became a firm international norm. Comparative advantage did not allow a small group of economies to monopolize the use of all available technologies at a time.

In the end, it is obvious that abundant labor is not a sufficient but only a necessary condition for a miracle. The availability of technologies and social capability for better adoption of them are crucial. Many authors have suggested that leadership, institution and policy did everything for a miracle.\textsuperscript{61} Consider-

\textsuperscript{58}Olson (1982, 1996) attempted to explain postwar economic growth distinct conversation from institutional perspective. Acemoglu, Johnson and Robinson (2005) develop the case that differences in economic institutions are the fundamental cause of differing economic development. On the other hand, the historical evidence of persistence of bad institutions and its consequences is well documented by Sokoloff and Engerman (2000).

\textsuperscript{59}See Abramovitz and David (1996), pp.50-51.

\textsuperscript{60}See p.389.

\textsuperscript{61}See, for example, Schuman (2009) and Olson (1982,1996).
ering the role of labor in miracles, the statement may not be true. However, it
cannot be denied that when an economy starts to develop, it needs hard currency,
technology import and institutions, for which strong leadership and right policies
are prerequisite.

Leadership wakes a hibernating miracle up. The waking up business is cru-
ial to understanding the differing growth performances in developing economies.
Although the Philippines and Korea in 1960 were similar in many respects as
Lucas (1993) vividly compared, Korea’s standard of living has improved much
faster than the Phillipines’s. It is not easy to find an answer for the difference in
other than leadership, policy and institution in the end.

6. CONCLUSION

A model of economic growth with abundant labor and scarce capital is con-
structed. The urgent purpose of constructing the model is to understand better the
economic miracles which have been staged since the end of the Second World
War. The representative miracles right after World War II were of West Germany
and Japan. However, their patterns were distinct from each other. Japanese mira-
cle was the pattern of Lewis (1954), whereas that of East Germany was of Solow
(1956).

West Germany was an economy fully industrialized even before the Second
World War. However, due to both the destruction of capital stock during the
war and the massive influx of immigrants from Eastern Europe, West German
capital/labor ratio was far below the steady state after the war. The growth path
of an industrialized economy with capital/labor ratio below steady state is well
known. West German GDP per capita grew fast initially. However, the growth
pace slowed down gradually from the start toward its steady state.

By contrast, Japan was not fully industrialized by the time the war ended.
The primary sector employment and rural population were more than forty eight
and sixty percent in 1950 respectively. Furthermore, more than thirty percent
of industrial capital was destroyed by allied bombing. If real wage rate is fixed
due to abundant labor in an economy equipped with a constant returns to scale
technology, the marginal product of capital is constant and hence an endogenous
growth emerges. However, high speed growth is sustained only for a limited

62 The Philippines and Korea were ruled by dictators for the same span of years between 1972
and 1986. However, every dictatorial leadership has its own quality. Korean dictators, Park and
Chun, pursued economic growth vehemently, while Philippine dictator, Ferdinand Marcos, sought
capital flight. We see the consequences.
span of time until full utilization of labor. In sum, Japan’s era of high speed
growth is another expression of endogenous growth due to abundant labor and
industrialization.

Many miracles including East Asian Tigers, China, Indonesia and many of
the transition economies in Eastern Europe including Poland, Czech Republic,
Slovenia etc. have been rising around the globe since the latter half of the twen-
tieth century. Note that almost all of them are of Japanese pattern with varying
extent. An economic miracle is intrinsic to a modern economy in the sense that
people is its seed. However, having a seed is one thing, germinating it is another.

We can see so many economies of abundant labor struggling under poverty,
which is obviously due to the lack of saving in most of the cases. Most of the
underdeveloped economies face credit constraint in the world financial market
and hence a vicious circle of low investment, low per capita capital stock, low
labor productivity and low income. However, once an economy with abundant
labor achieves appropriate social capability, i.e. right leadership, institution, at-
titude and policies for acquiring the means of accumulating capital, it enters the
endogenous growth phase and the standard of living improves rapidly. New mir-
acle emerges.
REFERENCES


APPENDIX 1: DERIVATION OF EQUATIONS IN SECTION 4

With the assumption that \( h_t = h \) (fixed), we have the following first order conditions for working hours, weeks and investment.

\[
(1 - \theta)k_t^\theta (hne_t)^{1-\theta} \frac{1}{e_t} = \frac{B}{1 + \gamma} \tag{A.1.1}
\]

\[
\frac{1}{(c_t - B_1 e_t)^\sigma} = \frac{\beta \theta k_{t+1}^{\theta - 1} (hne_{t+1})^{1-\theta} + 1 - \delta}{(c_{t+1} - B_1 e_{t+1})^\sigma} \tag{A.1.2}
\]

The resource constraint now is the following.

\[
c_t + k_{t+1} = k_t^\theta (hne_t)^{1-\theta} \tag{A.1.3}
\]

Hence we can solve for the control rule for weeks from (A.1.1).

\[
e_t = \phi_1 k_t, \quad \phi_1 = \left( \frac{1 - \theta}{B_1} \right)^{\frac{\gamma}{\theta}} (hn)^{1-\theta}, \tag{A.1.4}
\]

However, this rule is only when \( 0 < e_t < 1 \). Hence the following condition has to be satisfied.

\[
e_t = \phi_1 k_t < 1 \iff k_t < \phi_t^{-1} \tag{A.1.5}
\]

Using (A.1.5), we can rewrite the output:

\[
y_t = \phi_2 k_t, \quad \phi_2 = (hn\phi_1)^{1-\theta} = \left[ \frac{(1 - \theta)hn}{B_1} \right]^{1-\theta}. \tag{A.1.6}
\]

Note in (A.1.6) that the marginal product of capital is not diminishing and economic growth takes place endogenously.

If \( \delta = 1 \), we can solve the equation system of (A.1.2) and (A.1.3) for consumption. Guess the rules for consumption and investment as follows.

\[
c_t = \mu_1 y_t, \quad k_{t+1} = (1 - \mu_1) y_t, \tag{A.1.7}
\]

\( \mu_1 \) is constant. Using (A.1.6) in (A.1.7), we have the following.

\[
c_t = \mu_1 \phi_2 k_t, \quad k_{t+1} = (1 - \mu_1) \phi_2 k_t \tag{A.1.8}
\]
Now we use (A.1.4) and (A.1.8) to have:
\[ c_t - B_1 e_t = (\mu_1 \phi_2 - B_1 \phi_1) k_t, \] 
(A.1.9)
and then use (A.1.8) and (A.1.9) in (A.1.3) to get:
\[ \mu_1 = (\beta \theta)^{1 - \sigma} \phi_2^{1 - \sigma} \left[ (1 - \theta) B_1 \right]^{\frac{1 - \theta (1 - \sigma)}{\delta \sigma}}. \] 
(A.1.10)
Hence the guess in (A.1.7) is correct.
If \( e_t = 1 \), (A.1.3) can be written as follows.
\[
\frac{1}{(c_t - B_1)^{\sigma}} = \frac{\beta \theta k_{t+1}^{\theta - 1} (h n e_{t+1})^{1-\theta} + 1 - \delta}{(c_{t+1} - B_1)^{\sigma}} 
\]
(A.1.3a)
With this preference, even the assumption that \( \delta = 1 \) does not allow us to solve analytically the simultaneous equation system of (A.1.3a) and (A.1.3) for \( c_t \) and \( k_{t+1} \). However, note that if \( e_t = 1 \), production takes place according to the following technology.
\[ y_t = (h n)^{1-\theta} k_t^{\theta} \] 
(A.1.11)
(A.1.11) contrasts sharply with (A.1.6). The marginal product of capital diminishes in (A.1.11) as capital accumulates, while it does not in (A.1.6). Hence once the economy reaches full employment, i.e. \( e_t = 1 \), it converges to a steady state along a saddle path.
APPENDIX 2: DERIVATION OF EQUATIONS IN SECTION 5

Assume the real wage rate is fixed and the following production function is homogeneous of degree one in capital and labor.

\[ Y = F(K_t, N_t) \]  \hspace{1cm} (A.2.1)

For the cost to be minimized, the following has to hold.

\[ w = \frac{\partial F}{\partial N_t}, \quad r_t = \frac{\partial F}{\partial K_t} \]  \hspace{1cm} (A.2.2)

Since the production function is homogeneous of degree one, the marginal products are homogeneous of degree zero and hence it is a function of only capital/labor ratio. The first condition in (A.2.2) implies that the capital/labor ratio is constant. If we use the constant capital/labor ratio in the second equation in (A.2.2), we have the rental price (marginal product) of capital which is also constant. Now we have the following from the production function.

\[ \lambda Y_t = F(\lambda K_t, \lambda N_t). \]  \hspace{1cm} (A.2.3)

Use \( \lambda = 1/K_t \) to have:

\[ \frac{Y_t}{K_t} = F \left( 1, \frac{N_t}{K_t} \right) \iff Y_t = \alpha_1 K_t, \quad \alpha_1 = F(1, m), \]  \hspace{1cm} (A.2.4)

where \( m = N_t/K_t \), which is constant. The equilibrium technology is a linear function of capital. The endogenous growth will be limited by the labor endowment.

\[ N_t = mK_t < \bar{N} \iff K_t < m \bar{N}. \]  \hspace{1cm} (A.2.6)

Suppose the production function is Cobb-Douglas:

\[ Y_t = K_t^\theta (h_t N_t)^{1-\theta} \]  \hspace{1cm} (A.2.7)

and real wage increases according to:

\[ w_t = w h_t^{1-\theta}. \]  \hspace{1cm} (A.2.8)

The cost minimization requires:

\[ w = (1-\theta) \left( \frac{K_t}{N_t} \right)^\theta, \quad r_t = \theta h_t^{1-\theta} \left( \frac{K_t}{N_t} \right)^{\theta - 1}. \]  \hspace{1cm} (A.2.9)
Hence the capital/labor ratio is constant.

\[
\frac{K_t}{N_t} = \left( \frac{w}{1 - \theta} \right)^{\frac{1}{\theta}}, \quad r_t = \theta h_t^{1-\theta} \left[ \frac{1-\theta}{w} \right]^{\frac{1}{\theta}} \quad \text{(A.2.10)}
\]

Using the capital/labor ratio in the production function, we have the following.

\[
Y_t = \left( \frac{1 - \theta}{w} \right)^{\frac{1-\theta}{\theta}} h_t^{1-\theta} K_t \quad \text{if} \quad K_t < \left( \frac{w}{1 - \theta} \right)^{\frac{1}{\theta}} N \quad \text{(A.2.11)}
\]
Figure 1: The Rise and Fall of Miracles

(a) Germany

(b) Japan

Figure 2: Employment in Primary Sector

Source: Statistisches Gesamt (Germany), Up to 1989, West Germany and United Germany since 1990. Japan Statistics Bureau (Japan), Historical data.
Figure 3: Growth Rate of GDP per capita After WWII

(a) US  
(b) UK  
(c) France  
(d) Italy  
(e) Former USSR  
(f) Spain

Figure 4: East Asian Tigers

(a) Hong Kong  
(b) Korea  
(c) Singapore  
(d) Taiwan

Source: Up to 2008 Maddison data set. Since 2009, OECD (Korea), Hong Kong Monetary Authority (Hong Kong), Taiwan National Statistics (Taiwan), and Statistics Singapore (Singapore).
Figure 5: Recent Miracles

(a) China  
(b) Indonesia  
(c) Russia  
(d) Poland  
(e) Czech Republic  
(f) Slovenia

Source: Up to 2008, Maddison data set. After 2008, OECD (China, Russia, Poland, Czech Republic, Slovenia), Indonesia Statistics (Indonesia).
Figure 6: Selection and Convergence: Prewar and Postwar

(a) Maddison’s 16
(b) Club A
(c) Club B
(d) Club C
(e) Switzerland
(f) Coefficient of Variation

Data: Maddison data set.
Table 1: WWII Casualties

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Source: Ellis (1995), Table 51.

Table 2: Urban Population

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<td>Oct., 1946</td>
<td>68.6</td>
</tr>
<tr>
<td>Sept., 1950</td>
<td>71.1</td>
</tr>
</tbody>
</table>

Source: Hirshleifer (1963), Table 22.