Journal of Economic Theory and Econometrics, Vol. 28, No. 1, Mar. 2017, 49–66

Long-run Dynamic Correlation of Nonstationary Variables When the Trends are Misspecified *

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Abstract We study long-run comovement of the nonstationary time series variables with a focus on the use of coherency, defined as the long-run dynamic correlation. We pay attention to the effect of specification of trends on the long-run correlations by analyzing the cases that the data are either correctly or incorrectly detrended. Our simulation studies show that when the true process is trend stationary, time-removed long-run correlation estimates perform well, whereas the differenced case fails to generate valid outcomes due to degeneracy of the spectrums at the zero frequency of the series. We also provide empirical applications using unemployment rates of major cities in Korea from 1999 to 2016, and exemplify that false detrending could lead to nocuous outcomes. This work brings attention to correct specification of trends in nonstationary economic data in practice.

Keywords stochastic trend, deterministic trend, long-run correlations, detrending, degeneracy.

JEL Classification C14, C22, E10

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*The author is grateful to two anonymous referees for insightful comments. Thanks also go to Seojin Jung for research assistance.

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Received February 18, 2017 , Revised March 17, 2017, Accepted March 18, 2017

1.INTRODUCTION

It has been fundamental and long-standing issue to correctly specify the trends in most of nonstationary time series (Nelson and Kang, 1981; Nelson and Plosser, 1982; Perron, 1989; Murray and Nelson, 2000, Perron and Wada, 2009, to name a few). As widely known in time series econometrics, the series having a stochastic trend is widely known as an integrated process with a possible drift, whereas the series with a deterministic trend consists of a stationary components around a time trend. As is well known, it is hard to distinguish the two different types of trends due to inevitably low power of conventional unit root tests including Augmented Dickey-Fuller(ADF) test, the LM test by Kwiatkowski et al(1992; KPSS hereafter) and so on.

Given this, one is often vulnerable to misspecification of trends in practice. Nelson and Kang(1981) analyze the effect of inappropriately detrended series on the behavior of spectrums in the low and high frequency in the context of long-established business cycle literatures. Recently, Dagum and Giannerini(2006), among others, analyze the effects of misspecification of trends on certain hypothesis tests-stationarity tests and nonlinearity tests. Through a large set of simulation studies, they found that false detrending severely impact the performance of the tests(e.g., size of the tests). Also, Ashley and Verbrugge(2006) also study the effects of false detrending on the parameter estimates in linear models. These findings reminds the importance of correct detrending in understanding the true dynamics of the nonstationary time series variables.

In this work, we study effects of detrending on the measure of comovement between the nonstationary time series. In doing so, we employ dynamic correlation(DC) measure proposed by Croux et al(2001) as an useful comovement measure. The DC is basically the real part of coherency, where the coherency is the widely-used correlation coefficient in the frequency domain. While the DC provides correlation at any frequency, we restrict our attention to the long-run comovement, say, the long-run DC(LDC), which is the value of the DC at the zero frequency. Croux et al(2008) deal with the relationship between the LDC and cointegration. Conversely, if the true processes are indeed trend stationary rather than random walks with a possible drift, it is underemphasized how false detrending affects the LDC. We formally study the behavior of the LDC by correct and incorrect detrending, together with linear structures of innovation processes of the trended variables.

We perform a small set of simulation studies to see the effects of detrending on the LDC measures. Through this simulation studies, it is expected to know the cost of misspecification of trends. We also conduct a short empirical application. In doing so, we use monthly unemployment rates of seven major cities in Korea from June 1999 to December, 2016. The ADF test results indicate that the unit root hypothesis is rather strongly rejected with a very few exception in the autoregressive model with a possible drift term. The linear time trend is found to be highly significant for all the cities. Given this finding, we compute the LDC using two different detrending-time removal detrending and first differencing. The current works continue to study the short-run comovements of the time series variables using the dynamic correlations at the high frequencies. Thus, it is closely related to identification of periodicity of the comovements in terms of long-established business cycle literature.

2. STOCHASTIC OR DETERMINISTIC TRENDS FOR LONG-RUN DYNAMIC CORRELATIONS

2.1. LONG-RUN DYNAMIC CORRELATION

The dynamic correlation, proposed by Croux et al(2001) is simply the real part of the coherency between the two covariance stationary processes x and y. It is regarded as the correlation measure in the frequency domain. As interestingly motivated in their work, the dynamic correlation provides an useful comovement measure, given by

$$\rho_{x,y}(\lambda) = \frac{f_{xy}(\lambda)}{\sqrt{f_x(\lambda) f_y(\lambda)}}, \text{ for } \lambda \in [-\pi, \pi], \tag{1}$$

where $f_{xy}(\lambda)$ is the real part of the cross spectral density, called as cospectrum, $f_x(\lambda)$ and $f_y(\lambda)$ denote the auto spectrum densities of x and y, respectively(Priestley, 1981). As the dynamic correlation in (1) is represented in the frequency domain, it can provide different implications from correlations defined in the time domain. Thus, unlike correlations between two different time points, the dynamic correlations measure frequency-based correlations of the two variables, say, short-run, medium-run or long-run correlations(Croux et al, 2001).

In this paper, we restrict our attention to the long-run relationship by fixing the frequency equal to zero, where the zero-frequency quantity refers to the sum of all the correlations between the two variables. The long-run dynamic correlation(LDC hereafter) is given by

$$\rho_{xy}(0) = \frac{f_{xy}(0)}{\sqrt{f_x(0) f_y(0)}}, \tag{2}$$

where cross and auto spectral densities at the origin are defined by

$$\begin{aligned} f_{xy}(0) &= \textstyle\sum_{j=-\infty}^{\infty} R_{xy}(j) \\ f_x(0) &= \textstyle\sum_{j=-\infty}^{\infty} R_x(j), \quad f_y(0) = \sum_{j=-\infty}^{\infty} R_y(j), \end{aligned} \tag{3}$$

and $R_{xy}(j), R_x(j)$, and $R_y(j)$ be the autocovariance between x and y, auto-covariances of x and y at the j-th lags, respectively. Note that in the cross-spectral density function, the cross covariances $R_{xy}(j)$ and $R_{xy}(-j)$ are not equal. Besides, the usual constant factor 2π or $(2\pi)^{1/2}$ in the definitions (2) is unnecessary, thus is omitted.

The relationship between the LDC and cointegration was studied in Croux et al(2001). It is summarized as follows. Let integrated series including random walk with a drift be s_t and d_t. Then, for the first-differenced series $x_t = \Delta s_t, y_t = \Delta d_t$, if the two series are cointegrated, the LDC equals to 1 or -1, thereby the squared LDC equals to 1. We can also provide additional explanation based on spectral representations(Priestley, 1981, ch.9). If the two series are cointegrated, then $s_t - \beta d_t = \alpha + \theta t + \varepsilon_t$,where non-zero β is a cointegrating parameter and ε_t

is covariance stationary error. Then, $f_{xy}(0) = \beta f_y(0)$ and $f_x(0) = \beta^2 f_y(0)$, where $f_{xy}(0), f_x(0), f_y(0)$ denote the cross-spectral density, auto spectral densities of x and y, respectively. Then, the squared LDC equal to one.

On the other hand, the behavior of the LDC is unknown in the presence of misspecification of trends. In other words, if the true series are trend stationary, then detrending by first-differencing is incorrect, whereas correct detrending comes from removal of the deterministic time trend component from the underlying nonstationary variables. Below, we analyze the effects of correct and incorrect detrending on the behavior of the LDC.

2.2. CORRECT DETRENDING

Suppose the true data generating processes consist of stationary components around the deterministic time trend. Stationary fluctuations are allowed to have a general linear structures, as in Phillips and Solo(1992). Formally, we put the following assumption.

Assumption 1: *Bivariate series* z_t and w_t *follow trend stationary processes,*

$$(i)\ z_t = a + bt + e_t, \quad w_t = c + dt + u_t,$$

where the innovations $\eta_t = (e_t, u_t)'$ *be the linear processes given by*

$$\begin{aligned}(ii)\ \eta_t &= \phi(L)\varepsilon_t = \textstyle\sum_{j=0}^{\infty} \phi_j \varepsilon_{t-j} = \begin{pmatrix} \phi^1(L) \\ \phi^2(L) \end{pmatrix} \varepsilon_t, \\ \textstyle\sum_{j=0}^{\infty} j||\phi_j|| &< \infty, \text{ for } ||\phi_j|| = [\textstyle\sum_n \sum\limits_m |\phi_j(n,m)|^p]^{1/p},\ p \geq 1, \\ \det \phi(z) &\neq 0 \text{ for all } |z| \leq 1.\end{aligned}$$

where $\phi^1(L), \phi^2(L)$ be the first and second row of $\phi(L)$ matrix and $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ be *iid* with $E(\varepsilon_{1t}) = E(\varepsilon_{2t}) = 0$ and $E(\varepsilon_{1t}^2) = \sigma_1^2 > 0, E(\varepsilon_{2t}^2) = \sigma_2^2 > 0$.

The linear structure of innovations is standard condition for linear processes, in relation to the law of large numbers or asymptotic normality, etc. See Phillips and Solo(1992) for modification of linear structures.

Under the linear structure of the innovations, we can derive the explicit form of the LDC, as follows.

Theorem 1: *Under the assumption 1, the long-run dynamic correlation equals to*

$$\rho_{xy}(0) = \frac{D}{\sqrt{C_1 C_2}},$$

where

$$\begin{aligned} D &= \sum_{j=-\infty}^{\infty}\left(\sum_{k=0}^{\infty}\phi_k^{11}\phi_{k-j}^{21}\sigma_1^2 + \sum_{k=0}^{\infty}\phi_k^{12}\phi_{k-j}^{22}\sigma_2^2\right), \\ C_1 &= \sum_{j=-\infty}^{\infty}\left(\sum_{k=0}^{\infty}\phi_k^{11}\phi_{k-j}^{11}\sigma_1^2 + \sum_{k=0}^{\infty}\phi_k^{12}\phi_{k-j}^{12}\sigma_2^2\right) \\ C_2 &= \sum_{j=-\infty}^{\infty}\left(\sum_{k=0}^{\infty}\phi_k^{21}\phi_{k-j}^{21}\sigma_1^2 + \sum_{k=0}^{\infty}\phi_k^{22}\phi_{k-j}^{22}\sigma_2^2\right). \end{aligned}$$

[proof] The proof is based on Phillips and Solo(1992, eq.28) and Maynard and Shimotsu(2009). First, for the $f_x(0)$ and $f_y(0)$, we write $\phi^1(L) = (\phi^{11}(L), \phi^{12}(L))$. Then

$$R_x(j) = \sum_{k=0}^{\infty}\phi_k^{11}\phi_{k-j}^{11}\sigma_1^2 + \sum_{k=0}^{\infty}\phi_k^{12}\phi_{k-j}^{12}\sigma_2^2 = C_j^{11}(1)\sigma_1^2 + C_j^{12}(1)\sigma_2^2,$$

where $C_j^{11}(L) = \sum_{k=0}^{\infty}\phi_k^{11}\phi_{k-j}^{11}L^k$ with the lag operator L, and $\sigma_1^2 = \sigma_{[1,1]}^2$, $\sigma_2^2 = \sigma_{[2,2]}^2$. Similarly, we obtain

$$R_y(j) = C_j^{21}(1)\sigma_1^2 + C_j^{22}(1)\sigma_2^2,$$

where $C_j^{21}(L) = \sum_{k=0}^{\infty}\phi_k^{21}\phi_{k-j}^{21}L^k$. It follows that $f_x(0) = \sum_{j=-\infty}^{\infty}R_x(j) = C^{11}(1)\sigma_1^2 + C^{12}(1)\sigma_2^2$, where $C^{11} = \sum_{j=-\infty}^{\infty}C_j^{11}$ and $C^{12} = \sum_{j=-\infty}^{\infty}C_j^{12}$. Likewise, $f_y(0) = C^{21}(1)\sigma_1^2 + C^{22}(1)\sigma_2^2$. Next, for the cospectrum, we use Maynard and Shimotsu(2009, lemma 13) to get

$$
\begin{aligned}
R_{xy}(j) &= tr[\sum_{k=0}^{\infty}(\phi_k^1)'\phi_{k-j}^2\sigma^2] \\
&= \sum_{k=0}^{\infty}\phi_k^{11}\phi_{k-j}^{21}\sigma_1^2 + \sum_{k=0}^{\infty}\phi_k^{12}\phi_{k-j}^{22}\sigma_2^2 \\
&= D_j^1\sigma_1^2 + D_j^2\sigma_2^2.
\end{aligned}
$$

Thus, the numerator of the LDC equals to $f_{xy}(0) = D^1(1)\sigma_1^2 + D^2(1)\sigma_2^2$, where $D^1 = \sum_{j=-\infty}^{\infty} D_j^1$ and $D^2 = \sum_{j=-\infty}^{\infty} D_j^2$.

In relation to the Theorem1, we provide several remarks.

Remarks 1. The LDC of trend stationary processes depends on the magnitude of correlation between the innovations, given by the structure of the ϕ matrix.

Remarks 2. If the two innovations are uncorrelated, then the LDC $\rho_{xy}(0) = 0$, as is well expected. When e_t and u_t are uncorrelated, then the $\phi(L)$ becomes a diagonal matrix, where $\phi_k^{12} = \phi_k^{21} = 0$ for $k = 0, \pm 1, \pm 2, \ldots$ Thus, in is inferred that $D_j^1 = D_j^2 = 0$, which leads to $f_{xy}(0) = 0$ as well as $\rho_{xy}(0) = 0$.

2.3. INCORRECT DETRENDING

Next, we turn to the effect of inappropriate detrending. In time series context, Nelson and Kang(1981), among others, analyze the effect of false detrending when the true process is a random walk. They compare the behavior of autocorrelations and sample spectrum both in the low and high frequencies and show that incorrectly detrended variables cause misleading inferences including spurious periodic patterns in the data.

In this work, we pay attention to the possibility that trend stationary series are inappropriately first-differenced, which is the converse case of Nelson and Kang(1981). This phenomenon is well known as the moving average(MA) unit root(e.g., Saikkonen and Luukkonen, 1993). Also, in the view of frequency domain approach, the MA unit roots cause the spectral density of the series at the origin to become zero(Lee, 2010), in other words, $f_x(0) = 0$ or $f_y(0) = 0$ or both.

For example, since $x_t = \Delta e_t$, we compute $R_x(0) = 2\sigma^2$, $R_x(1) = R_x(-1) = -\sigma^2$, and $R_x(j) = 0$, for $|j| > 1$. Then, $f_x(0) = 0$. Then, by Cauchy-Schwarz inequality, $f_{xy}^2(0) \leq f_x(0)f_y(0)$, the cospectrum $f_{xy}(0)$ becomes zero. It follows that the LDC defined as in (2) takes indeterminate form as it takes the ratio of zero to zero. Thus, the LDC is invalid in the case of false detrending. We here note that the inference is extendable to the multivariate case of the cohesion measure, which is weighted dynamic correlations, proposed by Croux et al(2001). If some of the LDCs are invalid due to degeneracy, then the cohesion become also invalidated. Our empirical examples below clearly show the misleading outcome due to false detrending of the trend stationary variables.

As a digression, we analyze the squared LDC at the zero frequency, under degeneracy of spectrums, by the repeated use of L'Hospital's rule,

$$\text{squared } LDC(\lambda) = \frac{[f_{xy}''(0)]^2}{f_x''(0)f_y''(0),}, \text{ for } \lambda \to 0, \tag{4}$$

where the second derivatives of cross and auto spectrum at the zero frequency are assumed to be strictly positive. Note that the second derivatives are closely related to the smoothness of the spectral density function at the origin, where the second derivative as $f_x''(0) = -\sum_{j=-\infty}^{\infty} |j|^2 R_x(j)$, (Andrews, 1991). Other quantities $f_y''(0)$ and $f_{xy}''(0)$ are similarly defined. We note that estimation of the second derivatives involving sample variance times squared lags tends to be much more sensitive to the choice of lag truncation numbers than estimation of spectrum itself.

2.4. ESTIMATION OF LONG-RUN DYNAMIC CORRELATIONS

The LDC are estimable by conventional nonparametric estimation of spectral densities. For the auto-spectral densities of x and y, we have

$$\hat{f}_x(0) = \hat{R}_x(0) + 2\sum_{j=1}^{T-1} k(j/M)\hat{R}_x(j), \quad \hat{f}_y(0) = \hat{R}_y(0) + 2\sum_{j=1}^{T-1} k(j/M)\hat{R}_y(j), \tag{5}$$

where k is a kernel function, M is the lag truncation number and the sample variances are given by

$$\hat{R}_x(j) = T^{-1}\sum_{t=|j|+1}^{T}(x_t - \bar{x})(x_{t-|j|} - \bar{x}), \hat{R}_y(j) = T^{-1}\sum_{t=|j|+1}^{T}(y_t - \bar{y})(y_{t-|j|} - \bar{y}),$$

and $\bar{x}, \bar{y}$ are the sample means of x and y. The cospectrum estimator is given by

$$\begin{aligned} \hat{f}_{xy}(0) &= \sum_{j=1-T}^{T-1} k(j/M)\hat{R}_{xy}(j) \\ &= \hat{R}_{xy}(0) + \sum_{j=1}^{T-1} k(j/M)\hat{R}_{xy}(j) + \sum_{j=1}^{T-1} k(j/M)\hat{R}_{xy}(-j), \end{aligned} \tag{6}$$

where the sample cross covariance is equal to

$$\hat{R}_{xy}(j) = T^{-1}\sum_{t=|j|+1}^{T}(x_t - \bar{x})(y_{t-|j|} - \bar{y}),$$

and cross covariance is not symmetric in j.

One can use various types of kernel functions such as truncated, Bartlett, Gaussian, Quadratic spectral kernels, etc. The bandwidth M needs to be chosen to satisfy the condition that $M \to \infty$ and $M/T \to 0$, which guarantees consistency of the kernel-based estimators(e.g., Priestley, 1981, Andrews 1991, Newey and West 1994). However, we do not further a discussion for the bandwidth selection in detail, as it belongs to context of heteroskedasticity and autocorrelation consistent (HAC) covariance estimation which deviates from the focus of our work.

3. SIMULATION STUDIES

In this section, we conduct a small set of Monte Carlo simulations to see the effect of correct and incorrect detrending on the LDC. In doing so, we consider a linear time trend process as the data generating process. Let $Q_t = (x_t, y_t)'$ follow

$$Q_t = \alpha + \beta t + \eta_t, \tag{7}$$

where $\alpha = (\alpha_1, \alpha_2)', \beta = (\beta_1, \beta_2)'$, and $\eta_t = (e_t, u_t)'$, where each comes from $N(0,1)$, with $E(e_t) = E(u_t) = 0, E(e_t u_t) = \sigma_{eu}$ and $E(e_t u_s) = 0$ for $t \neq s$.

Correlation of innovations σ_{eu} are set from -0.5 to 0.5. Also, we set $\alpha = 0$, and $\beta = 0.1$ without loss of generality. The true LDC value, after correct detrending, $\rho_{xy}(0) = f_{xy}(0)[f_x(0)f_y(0)]^{-1/2} = \sigma_{eu}$. The sample sizes $T = 200$ and 500 are considered. We run 1,000 iterations to compute the mean squared error(MSE) as well as the sample mean of the LDC estimates. For estimation of the LDC, truncated kernel is used for simplicity. Bartlett kernels yielded similar results, though. The lag truncation number is selected by the proposed method in Newey and West(1994), where $M = [4(T/100)^{2/9}]$, and $[z]$ is the closest integer to z. This choice of lag truncation ensure the consistency of the estimator as the bandwidth grows at a slower rate than the sample size and, most importantly, is easy to implement. For reference, Croux et al(2001) chose fixed bandwidth.

As stated above, false detrending of differencing causes degenerate values of the denominators in the LDC, thus it fails to produce real-valued LDC as much as about 50% out of 1,000 replications in the simulations. Thus, we only report the MSE and average LDC values by correct detrending of time removal in the Table 1. It is found that the nonparametric estimates of the LDC performs quite well in terms of the bias(difference between the average LDC value and the true value) of the LDC estimates. The MSE is reduced almost by half as the sample size increase to 500, where the bias remains nearly unaffected. This result is expected since the bias of the spectral density estimator at the zero frequency only depends on the bandwidth, whereas the variance decreases with the sample size(e.g., Andrews, 1991).

4. EMPIRICAL STUDIES

We apply the inference to the real data. For a good candidate of a deterministic trend process, we choose monthly unemployment rates of major seven cities in Korea- Seoul, Busan, Incheon, Daejeon, Daegu, Gwangju and Ulsan, from June 1999 to December 2016. Data is obtained from Korean Statistical Information Service(KOSIS). As a preliminary work, we conducted the unit root tests. The table 2(a) reports the test results of stationarity for the series. We consider augmented Dickey-Fuller(ADF) test and the nonparametric variance ratio test proposed by Breitung(2002). The ADF test values are obtained in the AR model with the linear trend. For whitening the residuals, 3 to 9 lags of the first differenced series are allowed. The unit root hypothesis is clearly rejected at the 5% significance level, except for several cases. Next, we conducted the variance ratio test, defined as the ratio of $T^{-2}\sum\hat{S}_t^2$ and $\sum\hat{u}_t^2$, where $\hat{u}_t$ is the residual from a regression of the series on an intercept and a linear time trend and $\hat{S}_t$ is the partial sums, $\hat{S}_t = \hat{u}_1 + \hat{u}_2 + ... + \hat{u}_t$. The test has nonstandard limit distribution with the null hypothesis of unit roots(I(1)). The left-tailed critical values are given through simulations by Breitung(2002). It is found that the unit root hypothesis is rejected at the 5% level for Seoul, Daejeon, Daegu and Ulsan. On the other hand, the case of Busan supports the unit root hypothesis, and the case of Gwangju seems a bit ambiguous.

In sum, these test results imply that the unemployment rate of the major city is likely to be trend stationary process, though there exists a few exceptions. This finding is different from Kim et al(2012), who present the evidence for difference stationarity for the unemployment rates during 1994 and 2002. In an subsequent analysis, the table 2(b) shows the t-test values for the hypothesis of the linear time trend term is zero. It unexceptionally shows that the linear trend is significant at the 5% level, where the slightly downward sloping time trend is found during the given sample period. Besides, as a possible limitation in this empirical work, we do not explicitly deal with the issue of seasonality in our analysis. If we extend the analysis from the long-run to short-run dynamic correlations, then the issue of seasonality draws a good deal of attention. We leave this a future research topic.

Next, the table 3 provides the estimated LDC values based on correctly detrended and incorrectly differenced series. The LDC and its squared values between Seoul and other six cities are reported. If the series are correctly detrended by time removal, the LDC shows around 0.6 to 0.7, except for the case between Seoul and Incheon, whose LDC amounts to about 0.9. A possible reason that a high degree of long-run correlations between Seoul and Incheon could come from proximity in distance. On the other hand, if the series are differenced, then the LDC generates nonsensical values, where many LDC values exceed one. The LDC between Seoul and Daejeon is even undefined, as computation fails to generate real values. As theoretically analyzed above, this result comes from degeneracy of the auto-spectrum at the origin of the data. In turn, the empirical finding could support that the unemployment rate follows a trend stationary process. In sum, our empirical results provide a clear example how false detrending affects the long-run comovement measure in practice.

5. CONCLUSION

We study long-run comovement of the nonstationary time series variables with a focus on the concept of coherency defined as the long-run dynamic correlation. We particularly focus on the effect of mis-specification of trends on the long-run correlation measure by analyzing the cases between correct detrending and incorrect detrending. Simulation studies show that when the true process is trend stationary, time-detrended LDC performs well, whereas the differenced LDC fails to generate valid outcomes due to degeneracy of the spectral densities at the zero frequency of the series. As for empirical applications, we estimate the long-run correlation between unemployment rate between Seoul and major cities in Korea. It is found that the unemployment rates turn out to be trend stationary during the period from 1999 to 2016. Detrending of time removal yields the reasonable LDC estimates, whereas first differencing yield invalid LDC estimates, which often exceed unity. Thus, it gives an example that false detrending could lead to nocuous results. This work bring attention to correct specification of trends in nonstationary data in terms of statistical adequacy.

Inferences given in this work could extend to related research works. One direction includes an extension of inferences to analysis of short-run comovement of the dynamic correlations at high frequencies. It would be useful to investigate how the detrending procedures affect the comovement measures, particularly in the short-run. Thus, it is closely related to correctly identify the periodicity of trended time series variables in terms of long-established literature on business cycles.

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APPENDIX

Table 1. Mean Squared Errors(MSE) of the correctly detrended LDC Estimates

MSE \ values of σ_{eu}	0	0.2	-0.2	0.5	-0.5
$T = 200$					
Average LDC estimates	-0.003	0.1937	-0.1943	0.4874	-0.4879
MSE	0.0509	0.0476	0.0477	0.0312	0.0321
$T = 500$					
Average LDC estimates	-0.0023	0.1932	-0.1974	0.4891	-0.4915
MSE	0.0243	0.0225	0.0228	0.0146	0.0147

(1) 1000 replications are conducted for the sample sizes $T = 200$ and 500. (2) The LDC denotes the long-run dynamic correlation. The σ_{eu} denote the value of covariance of the innovations. (3) The LDC estimates are obtained from truncated kernel-weighted periodograms where the bandwidth is chosen by the method in Newey and West(1994).

Table 2(a): Results of Stationarity Tests

(i) ADF Test; $H_0 : \beta = 0$ in the regression: $\Delta y_t = \alpha + \beta y_{t-1} + \theta t + \sum_{j=1}^{p} \delta_j \Delta y_{t-j} + e_t$							
Lags(p) \ City	Seoul	Busan	Incheon	Daejeon	Daegu	Gwangju	Ulsan
3	-5.87**	-4.03**	-5.65**	-7.48**	-5.67**	-4.73**	-3.58**
6	-5.59**	-3.28*	-4.64**	-6.3**	-4.63**	-3.69**	-3.06
9	-4.79**	-3.89**	-5.35**	-4.4**	-4.52**	-3.59**	-2.34
(ii) Variance Ratio Test							
	0.0026**	0.0096	0.005	0.0006**	0.004**	0.0027**	0.0048

(1) For ADF test, the *(**) denote the rejection of unit root hypothesis at the 5%(10%) level, where the asymptotic cv= -3.43(-3.13).

(2) For the variance ratio test, the *(**) denote rejection of unit root hypothesis at the 5%(10%) level, where the asymptotic cv= 0.00355(0.0045).

Table 2(b): Test results for the linear trend for unemployment rates of major cities in Korea from 1999.6-2016.12:

City	Seoul	Busan	Incheon	Daejeon	Daegu	Gwangju	Ulsan
t-value	-6.76*	-10.39*	-2.21*	-9.63*	-8.94*	-17.82*	-5.07*

(1) The t-value denotes the t-statistic for $H_0 : \beta = 0$ in the linear trend model, $y_t = \alpha + \beta t + e_t$. The * denotes the significance at the 5% level.

Table 3. Long-run dynamic correlations and its squared between Seoul and six other major cities in Korea.

LDC (squared LDC)		Correctly detrended	Incorrectly Differenced
Seoul-	Busan	0.6951 (0.4832)	1.8564 (3.4463)
	Incheon	0.9049 (0.8188)	2.6666 (7.1106)
	Daejeon	0.6193 (0.3835)	undefined
	Daegu	0.7783 (0.6057)	2.1511 (4.6272)
	Gwangju	0.6982 (0.4874)	2.3977 (5.7191)
	Ulsan	0.6307 (0.3978)	0.1328 (0.0176)

(1) Nonparametric estimates of LDC are computed as in sec.2.3.